SERMON 2013
Schedule of Talks

Saturday, April 13

8:00-8:15am	Coffee and light refreshments outside Norton 101
8:15-8:35am	**Andrew Ledoan**, University of Tennessee at Chattanooga. *Differences between consecutive primes*
8:45-9:05am	**Jesse Thorner**, Wake Forest University. *The Explicit Sato-Tate Conjecture and Densities Pertaining to Lehmer-Type Questions*
9:15-9:35am	**Paul Spiegelhalter**, University of Illinois at Urbana-Champaign. *Distributions of Sequences Modulo 1: The Good, the Bad, and the Ugly*
9:45-10:05am	**Lola Thompson**, University of Georgia. *How often is $\#E(\mathbb{F}_p)$ squarefree?*
10:05-10:30	Coffee break
10:30-11:20	**Rachel Pries**, Colorado State University. *Most curves with p-rank 0 are not supersingular*
11:30-11:50	**Anastassia Etropolski**, Emory University. *A local-global principle for Galois representations attached to elliptic curves*
12:00-2:00pm	Lunch
2:00-2:50pm	**Peter Sarnak**, Institute for Advanced Study/Princeton University. *Thin matrix groups and monodromy of the hypergeometric function*
3:00-3:20pm	**Josh Hiller**, Western Carolina University. *A Generalization of Fermat's Little Theorem To Nonsingular Integer Matrices With Integer Eigenvalues*
3:20-3:45pm	Coffee break
3:45-4:05pm	**Justin DeBenedetto**, Wake Forest University. *Quadratic Forms Representing All Primes*
4:15-4:35pm	**Carrie Finch**, Washington & Lee University. *Perfect power Riesel numbers*
4:45-5:05pm	**Paul Young**, College of Charleston. *Rational series for multiple zeta and log gamma functions*

Sunday, April 14

8:00-8:15am	Coffee and light refreshments outside Norton 101
8:15-8:35am	**Michael Mossinghoff**, Davidson College. *Googol it!*
8:45-9:05am	**Brian Sinclair**, University of North Carolina Greensboro. *Finding Polynomials with Given Okutsu Invariants*
9:15-9:35am	**Dan Yasaki**, University of North Carolina Greensboro. *On the cohomology of linear groups over imaginary quadratic fields*
9:45-10:05am	**Jon Grantham**, IDA/CCS. *Collecting primes with* $p^2 - 1$ *1163-smooth*
10:05-10:30am	Coffee break
10:30-10:50am	**Bobby Grizzard**, The University of Texas Austin. *Heights in infinite extensions of* $\mathbb{Q}$
11:00-11:20am	**Heekyoung Hahn**, Duke University. *Algebraic cycles and Tate classes on Hilbert modular varieties*
11:30-11:50am	**Larry Rolen**, Emory University. *Ramanujan's Mock Theta Functions*
12:00-12:20pm	**Duc Van Huynh**, University of Florida. *Enumerating* k*-isomorphism classes of finite extensions of the local field* $k((\pi))$

Andrew Ledoan, University of Tennessee at Chattanooga

Title: Differences between consecutive primes

Abstract: In 1976, Gallagher proved that the Hardy-Littlewood prime k-tuple conjecture implies that, for the primes up to x, the number of primes in the interval $(x, x + \lambda \log x]$ follows a Poisson distribution with mean λ, where λ is any fixed positive constant. Recently, Professor Daniel A. Goldston and I proved that the number of consecutive primes with difference $\lambda \log x$ has the Poisson distribution superimposed on the conjectured asymptotic formula for pairs of primes with difference $\lambda \log x$. In this talk, I will present an extension of Gallagher's theorem and more precise asymptotic formulas if λ approaches zero as x tends to infinity. In order to establish these asymptotic formulas, we also proved new singular series average results. (This talk is based on my joint work with Professor Daniel Goldston at San José State University.)

Jesse Thorner, Wake Forest University

Title: The Explicit Sato-Tate Conjecture and Densities Pertaining to Lehmer-Type Questions

Abstract: Let $f = \sum_{n=1}^{\infty} a(n)q^n \in S_k^{\text{new}}(\Gamma_0(N))$ be a normalized Hecke eigenform with N squarefree. For a prime p, define $\theta_p \in [0, \pi]$ to be the angle for which $a(p) = 2p^{(k-1)/2}\cos(\theta_p)$. Let $I = [\alpha, \beta] \subset [0, \pi]$, and let $\mu_{ST}(I) = \int_\alpha^\beta \frac{2}{\pi}\sin^2(\theta)\, d\theta$ be the Sato-Tate measure. We prove, assuming that the symmetric power L-functions of f are automorphic and satisfy the Generalized Riemann Hypothesis, that

$$|\#\{p \in [x, 2x] : \theta_p \in I\} - (\pi(2x) - \pi(x))\mu_{ST}(I)| = O\left(\frac{x^{3/4}\log(Nkx)}{\log(x)}\right),$$

where the implied constant is $\frac{2\sqrt{15}}{3}$. This bound decreases by a factor of $\sqrt{\log(x)}$ if we let $I = [\frac{\pi}{2} - \frac{1}{2}\Delta, \frac{\pi}{2} + \frac{1}{2}\Delta]$, where Δ is small. This allows us to compute lower bounds for the density of positive integers n for which $a(n) \neq 0$. In particular, we prove that if τ is the Ramanujan tau function, then

$$\lim_{x\to\infty} \frac{\#\{n \in [1, x] : \tau(n) \neq 0\}}{x} > 1 - 8.8 \cdot 10^{-6}.$$

Paul Spiegelhalter, University of Illinois at Urbana-Champaign

Title: Distributions of Sequences Modulo 1: The Good, the Bad, and the Ugly

Abstract: The theory of distribution of sequences modulo 1 originated in the early 20th century with work by Weyl, Fejer, and van der Corput. It has since grown to an important branch of number theory with applications to many other areas such as numerical analysis and probability. The focus of nearly all of this development has been on the case of "nice" or "uniform" distribution, which roughly means that each subinterval of $[0, 1]$ contains its "proper" share of the terms of the sequence reduced modulo 1. In this survey we investigate the opposite case, that of sequences that are, in some sense, as far from uniformly distributed as possible, and we develop a cohesive theory of "bad" distribution. We consider several natural notions of "bad" distribution, each obtained by taking a well-known characterization of uniform distribution and turning it on its head, and study the relations between these notions. We tie together the few scattered and little known results that exist in the literature on the subject of bad distribution, produce a unified treatment, and round out the picture with some new results.

Lola Thompson, University of Georgia

Title: How often is $\#E(\mathbb{F}_p)$ squarefree?

Abstract: Let E be a non-CM elliptic curve defined over $\mathbb{Q}$. For each prime p of good reduction, E reduces to a curve E_p over the finite field $\mathbb{F}_p$ with $\#E_p(\mathbb{F}_p) = p + 1 - a_p(E)$, where $|a_p(E)| \leq 2\sqrt{p}$. In this talk, we discuss the problem of determining how often $\#E(\mathbb{F}_p)$ is squarefree. Our results in this vein are twofold. For any fixed curve E, we give an upper bound for $\pi_E^{SF}(X) :=$ the count of primes $p \leq X$ for which $p+1-a_p(E)$ is squarefree. We also provide evidence of an asymptotic for π_E^{SF} on average over all curves E in a suitable "box." This talk is based on joint work with Shabnam Akhtari, Chantal David and Heekyoung Hahn.

Rachel Pries, Colorado State University

Title: Most curves with p-rank 0 are not supersingular

Abstract: An elliptic curve defined over a finite field of characteristic p can be ordinary or supersingular; this distinction measures certain properties of its p-torsion. For the Jacobian of a curve X of genus $g > 1$, there are several ways to generalize this difference. The p-rank, Newton polygon, a-number, and Ekedahl-Oort type are invariants of the p-torsion of Jac(X). I will explain these invariants and show to compute them in SAGE. Then I will discuss how to prove the existence of curves with given p-torsion invariants using the geometric structure of the boundary of the p-rank strata of the moduli space of curves. J. Achter and I used this method to prove, in particular, that a generic (hyperelliptic) curve of genus g and p-rank 0 is not supersingular if $g > 2$.

Anastassia Etropolski, Emory University

Title: A local-global principle for Galois representations attached to elliptic curves

Abstract: Let E be an elliptic curve over a number field K. A result of Katz says that if E has a torsion point locally at almost every prime, then it does not necessarily have a torsion point over K, but it is isogenous to an elliptic curve with a K torsion point. Recently, Sutherland proved a local-global principle for isogenies which says that if E over Q admits an isogeny of prime degree locally almost everywhere, then it admits one over Q of the same degree, with the exception of a single counterexample at the prime 7. Both of these results can be restated in terms of images of Galois representations. For example, admitting an isogeny of degree p is equivalent to the image of the mod p Galois representation attached to E being contained in a Borel subgroup. We consider the most general formulation of this problem, i.e. for which subgroups G does the image locally being contained in G assure that it will be globally contained in G? If such a principle does not hold, what counterexamples exist?

Peter Sarnak, Institute for Advanced Study/Princeton University

Title: Thin matrix groups and monodromy of the hypergeometric function

Abstract: Thin matrix groups are groups of integer matrices which are infinite index in the Z-points of their Zariski closure. They come up in diophantine and geometric problems and in particular as mondromy groups. We describe some of the theory concentrating on the monodromy of the hypergeometric group.

Josh Hiller, Western Carolina University

TITLE: A Generalization of Fermat's Little Theorem To Nonsingular Integer Matrices With Integer Eigenvalues

ABSTRACT: We make use of recent results regarding Pascals matrices to generalize Fermats Little Theorem to invertible integer matrices with integer eigenvalues. We then propose a conjecture that would further expand on our results.

Justin DeBenedetto, Wake Forest University

TITLE: Quadratic Forms Representing All Primes

ABSTRACT: Building on the method used by Bhargava to prove "The Fifteen Theorem," we show that every integer valued positive-definite quadratic form which represents all prime numbers must also represent 205. We further this result by proving that 205 is the smallest nontrivial composite number which must be represented by all such quadratic forms.

Carrie Finch, Washington & Lee University

TITLE: Perfect power Riesel numbers

ABSTRACT: In 1956, Riesel showed that there are infinitely many odd positive integers k such that $k \cdot 2^n - 1$ is composite for all natural numbers n. The smallest known example is 509203. In 2003, Chen showed that for a fixed odd positive integer r, there are infinitely many odd positive integers k such that k^r is Riesel. Moreover, Chen conjectured that this result holds for all positive integers r. In 2006, Filaseta *et al* found examples of infinitely many positive integers k such that k^4 is Reisel, and other examples such that k^6 is Riesel. In recent work with Lenny Jones, we show that infinitely many odd positive integers k exist with k^r Riesel, with the constraint that $gcd(r, 105) = 1$.

Paul Young, College of Charleston

TITLE: Rational series for multiple zeta and log gamma functions

ABSTRACT: We give series expansions for the Barnes multiple zeta functions in terms of rational functions whose numerators are complex-order Bernoulli polynomials, and whose denominators are linear. We also derive corresponding rational expansions for Dirichlet L-functions and multiple log gamma functions in terms of higher order Bernoulli polynomials. These expansions naturally express many of the well-known properties of these functions. As corollaries many special values of these transcendental functions are expressed as series of higher order Bernoulli numbers.

Michael Mossinghoff, Davidson College

TITLE: Googol it!

ABSTRACT: A *Barker sequence* is a finite sequence of integers, $a_0, \ldots, a_{n-1}$, each ± 1, whose aperiodic autocorrelations are uniformly small: for each $k > 0$, one requires that $\left|\sum_{i=0}^{n-1-k} a_i a_{i+k}\right| \leq 1$. Very few Barker sequences are known, and it has long been conjectured that no additional ones exist. Many arithmetic restrictions have been established that severely limit the allowable lengths of Barker sequences, so severely

that no permissible lengths were even known. We identify the smallest plausible value for the length of a new Barker sequence, and we compute a number of permissible lengths up to a sizable bound (Google it!).

Brian Sinclair, University of North Carolina Greensboro

TITLE: Finding Polynomials with Given Okutsu Invariants

ABSTRACT: The OM Algorithm (named for both pairings of Ore-Mac Lane and Okutsu-Montes) is a highly effective algorithm for computing integral bases, factoring local field polynomials, ideal decomposition, and other related applications. It has been shown more recently that the intermediate values of the algorithm produce certain invariants that were first explored by Kousaku Okutsu in 1982 in relation to computing integral bases. In this presentation, we will summarize the process of the OM algorithm and explore the research into leveraging it to find polynomials with certain of these Okutsu invariants.

Dan Yasaki, University of North Carolina Greensboro

TITLE: On the cohomology of linear groups over imaginary quadratic fields

ABSTRACT: Let F be an imaginary quadratic field, and let O be its ring of integers. In this talk, I will report on recent computations of cohomology of $GL_n(O)$ for $n = 3, 4$ and for a selection of discriminants. In particular, we compute the integral cohomology up to p-power torsion for small primes p. Our main tool is the polyhedral reduction theory developed by Ash and Koecher generalizing work of Voronoi. Our results extend work of Staffeldt, who treated the case n=3 discriminant -4. This is joint work with Dutour, Gangl, Gunnells, Hanke, and Schurmann conducted as part of a "SQuaRE" (Structured Quartet Research Ensemble) at the American Institute of Mathematics.

Jon Grantham, IDA/CCS

TITLE: Collecting primes with $p^2 - 1$ 1163-smooth

ABSTRACT: The problem of finding a number that is simultaneously a Fermat pseudoprime base 2 and a Fibonacci pseudoprime is commonly referred to as the $620 problem. I give some mathematical and historical background. New techniques bring us closer to finding such a number, but we are still very far away.

Bobby Grizzard, The University of Texas Austin

TITLE: Heights in infinite extensions of $\mathbb{Q}$

ABSTRACT: We say a subfield of $\overline{\mathbb{Q}}$ has the Northcott property if it has only finitely many points of height $< T$, for all $T > 0$, and we say it has the Bogomolov property if there is a positive ϵ such that it has no element of positive height $< \epsilon$. We will discuss some recent results on infinite extensions of $\mathbb{Q}$ that satisfy these properties, as well as some basic questions that remain open.

Heekyoung Hahn, Duke University

TITLE: Algebraic cycles and Tate classes on Hilbert modular varieties

ABSTRACT: Let $E/\mathbb{Q}$ be a totally real number field that is Galois over $\mathbb{Q}$, and let π be a cuspidal, nondihedral automorphic representation of $\mathrm{GL}_2(\mathbb{A}_E)$ that is in the lowest weight discrete series at every real

place of E. The representation π cuts out a "motive" $M_{\text{ét}}(\pi^\infty)$ from the ℓ-adic middle degree intersection cohomology of an appropriate Hilbert modular variety. If ℓ is sufficiently large in a sense that depends on π we compute the dimension of the space of Tate classes in $M_{\text{ét}}(\pi^\infty)$. Moreover if the space of Tate classes on this motive over all finite abelian extensions k/E is at most of rank one as a Hecke module, we prove that the space of Tate classes in $M_{\text{ét}}(\pi^\infty)$ is spanned by algebraic cycles. This is joint work with J. R. Getz.

Larry Rolen, Emory University

Title: Ramanujan's Mock Theta Functions

Abstract: In his famous deathbed letter, Ramanujan introduced the notion of a mock theta function, and he offered some alleged examples. Recent work by Zwegers has elucidated the theory encompassing these examples. They are holomorphic parts of special harmonic weak Maass forms. Despite this understanding, little attention has been given to Ramanujan's original definition. Here we prove that Ramanujan's examples do indeed satisfy his original definition.

Duc Van Huynh, University of Florida

Title: Enumerating k-isomorphism classes of finite extensions of the local field $k((\pi))$

Abstract: Let K be a local field of characteristic p with a perfect residue field k. Let K_s be be a separable closure of K. Let $\mathcal{E}$ be the set of all totally ramified field extensions of K of degree n . In this paper, we wish to enumerate the k-isomorphism classes of $\mathcal{E}$.