Quiz 4, Math Thought

Dr. Graham-Squire, Spring 2016

5 min

Name: Key

1. (3 points) To prove that $(A \cup B) - B = A - (A \cap B)$, you would need to prove two set inclusions. Prove one of the set inclusions (whichever you prefer) for this problem.

($\subseteq$) ✓ Let $x \in (A \cup B) - B$. Then $x \in A \cup B$ ✓ and $x \notin B$ by def of set difference. $x \in A \cup B \Rightarrow x \in A$ ✓ or $x \in B$. But since $x \notin B$, we know that $x \in A$! ✓ Since $x \notin B$, we must have $x \notin A \cap B$ ✓, since if $x \in A \cap B$ then $x \in B$. Thus we conclude $x \in A$ and $x \notin A \cap B$ ✓

$\Rightarrow x \in A - (A \cap B)$ by def.

2. (2 points) Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5\}$. Use the definition of Cartesian product to explain why $A \times B \neq B \times A$.

$(1, 5) \in A \times B$, but $1 \notin B$, thus $(1, 5) \notin B \times A$

Since $A \times B$ and $B \times A$ have an element which is not common, they are not equal.

3. (2 points) State if the equation is True or False and justify your conclusions (do not necessarily need a full proof).

Let A, B and C be sets. If $A \cap C = B \cup C$, then $A = B$.

False. Suppose $A = \{1, 2\}$

$C = \{1, 2\}$

and $B = \{1\}$

Then $A \cap C = \{1, 2\} = B \cup C$

But $A \neq B$

4. (3 points) Use the choose-an-element method and definitions to carefully prove that $A \cap B^c \subseteq A - B$ (Note: they are in fact equal, but you do NOT need to prove the other set inclusion).

Let $x \in A \cap B^c$. Then $x \in A$ and $x \in B^c$ by def. of intersection. Then $x \notin B$ by def of complement. Since $x \in A$ and $x \notin B$, $x \in A - B$ by def of set difference.