Quiz 3, Math Thought

Dr. Graham-Squire, Spring 2016

Name: ________________________________

1. (5 points) Choose one of the following to prove using induction:

   (a) $2 + 5 + 8 + \cdots + (3n-1) = \dfrac{n(3n+1)}{2}$

   (b) $\dfrac{1}{1\cdot 2} + \dfrac{1}{2\cdot 3} + \cdots + \dfrac{1}{n(n+1)} = \dfrac{n}{n+1}$

2. (2 points) Evaluation of proof: The following induction proof has at least one mistake. Explain what it is (or they are). You do not need to check the algebra, it is definitely correct–you should be looking for mistakes in the structure of the proof:

Proof that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$, by induction:

Need to prove that $P(k+1)$ is true, which is the statement

$$1 \cdot 2 + 2 \cdot 3 + \cdots + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}.$$

Using the induction hypothesis on the left-hand side and simplifying the right-hand side we have:

$$1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$\Rightarrow 1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) = \frac{(k^2+3k+2)(k+3)}{3} - (k+1)(k+2)$$

$$\Rightarrow 1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) = \frac{k^3+6k^2+11k+6}{3} - \frac{3(k^2+3k+2)}{3}$$

$$\Rightarrow 1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) = \frac{k^3+3k^2+2k}{3}$$

$$\Rightarrow 1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) = \frac{k(k+1)(k+2)}{3}$$

which is $P(k)$ so this verifies the proof.

3. (3 points) Consider the sequence $a_1, a_2, \ldots, a_n, \ldots$ determined recursively as follows: $a_1 = 1$, $a_2 = 1$, and for each n we have the recursive definition $a_{n+2} = a_{n+1} + 3a_n$.

   (a) Calculate $a_1,\ a_2, \ldots, a_7$.

   (b) Make a conjecture about which of the entries in the sequence are even numbers.

   (c) Set up an induction proof for your conjecture. You do NOT have to actually prove it, just set up and explain what you would need to do to prove it.