Quiz 2, Math Thought

Dr. Graham-Squire, Spring 2016

8:59
9:04
5 ⇒ 15→20 min.

Name: Key

1. (4 points) Prove that if $x^3 - 7$ is irrational, then x is irrational.

Proof by contrapositive: Assume x is rational, want to prove that $x^3 - 7$ is rational. ✓

- Suppose x is rational. Then $x = \frac{p}{q}$ ✓ for some $p, q \in \mathbb{Z}$, $q \neq 0$. Then

$$x^3 - 7 = \left(\frac{p}{q}\right)^3 - 7$$
$$= \frac{p^3}{q^3} - \frac{7q^3}{q^3} \quad ✓$$
$$= \frac{p^3 - 7q^3}{q^3}$$

Since integers are closed under multiplication and subtraction, ✓ and $q \neq 0$, we have $p^3 - 7q^3 \in \mathbb{Z}$ and $q^3 \neq 0$, $q^3 \in \mathbb{Z}$.

Thus $x^3 - 7 = \frac{\text{integer}}{\text{integer}}$ and is rational by definition. ∎

2. (3 points) Let a and n be positive integers. Prove that if n divides a, then $a \equiv 0 \pmod{n}$. (Recall that $a \equiv b \pmod{n} \leftrightarrow n$ divides $a - b$).

Suppose $n | a$. Then $a = nk$ for some $k \in \mathbb{Z}$.

$$\Rightarrow a = 0 + nk$$

$$\Rightarrow a - 0 = nk$$

$\Rightarrow n$ divides $(a-0)$ by def. of divides

$\Rightarrow a \equiv 0 \pmod{n}$ by def. of $\equiv$.

3. (3 points) Consider the following statement: "For all integers x, there exists a positive real number y such that $3x < \sqrt{y}$."

(a) Write the statement above using only mathematical notation (or as much math notation as you can).

(b) Write, in words, the *negation* of the statement above.

1.5

(a) $(\forall x \in \mathbb{Z})\ (\exists y \in \mathbb{R}^+)\ (3x < \sqrt{y})$

negation is $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{R})\ (3x \geq \sqrt{y})$

1.5 (b) There exits an integer x such that for all positive real numbers y we have $3x \geq \sqrt{y}$.