Quiz 1, Math Thought

Dr. Graham-Squire, Spring 2016

8:57
9:04
7 ⇒ 20 min

Name: Key

1. (5 points) Let a *pogo* integer be an integer of the form $5n+1$ for some $n \in \mathbb{Z}$, a *bird* integer be of the form $5n+2$ for some $n \in \mathbb{Z}$, and a *dog* integer be of the form $5n+4$ for some $n \in \mathbb{Z}$. Prove that a dog integer plus a bird integer will always be equal to a pogo integer.

Let m be a dog integer and n be a bird integer.

WTF: m+n is a pogo integer.

Proof: If m is a dog integer, then $m = 5q+4$ for some $q \in \mathbb{Z}$, and if n is a bird integer then $n = 5p+2$ for some $p \in \mathbb{Z}$. Then $m+n = (5q+4)+(5p+2)$

$= 5q+5p+5+1$ ← C&C, $6 = 2+4 = 5+1$ and associativity

$m+n = 5(q+p+1)+1$ ← factoring

So m+n is a pogo integer b/c it is of the form $5l+1$, for $l = q+p+1$. ∎

−0.5 if use n for both

−0.5 for small calculation error

1.5

2. (3 points) Are the following two statements logically equivalent, negations of each other, or neither? Justify your answer. 1.5

$(P \to Q) \to Q$ and $\neg(Q \vee \neg P) \vee Q$

P	Q	$P \to Q$	$\neg P$	$Q \vee \neg P$	$\neg(Q \vee \neg P)$	$(P \to Q) \to Q$	$\neg(Q \vee \neg P) \vee Q$
T	T	T	F	T	F	T	T
T	F	F	F	F	T	T	T
F	T	T	T	T	F	T	T
F	F	T	T	T	F	F	F

logically equivalent b/c same.

3. (2 points) Use both (a) the roster method and (b) set builder notation to specify the set "All odd integers less than 50 which are divisible by 3." (Note: it is okay to use ... in your set, but it should be used in a way such that the pattern is clear.)

positive

(a) $\{3, 9, 15, \ldots, 39, 45\}$ ✓

(b) $\{x \in \mathbb{Z}^+ \mid (x < 50) \wedge (\exists n \in \mathbb{Z} \mid x = 2n+1) \wedge \left(\frac{x}{3} \in \mathbb{Z}\right)\}$ ✓

Allow one small mistake.