Research Statement

Dr. Adam Graham-Squire

In graduate school, I studied algebra, and my research investigated local formal integral transforms. As the years have passed, I have moved away from abstract algebra research to pursue interests more closely connected to real-world problems. My current academic research spans a variety of areas, the most active being voting theory (specifically ranked-choice voting), gerrymandering, recreational mathematics, and the Scholarship of Teaching and Learning (SOTL). I have also worked with 7 different students over the last 8 years in mentoring undergraduate research projects, one of which led to a publication. I have published in journals with focuses on mathematics, computational political science, and math education. I will briefly discuss my experience in each of the research areas described above, and then close with some thoughts about my general philosophy toward research.

Ranked-Choice Voting (RCV)

When there are more than two candidates in an election and voters are allowed to rank candidates in their vote, the increased amount of information creates the potential for more democratic elections than traditional means of voting. Mathematically, however, Arrow's Impossibility Theorem famously demonstrated that no election method is perfect ([1]). My research investigates both applied and theoretical aspects of RCV. On the applied side, I look at empirical data from Instant-Runoff elections in municipalities in the United States, particularly the Bay Area in California, to identify the frequency of voting anomalies such as monotonicity (a monotonicity anomaly is a situation in which a winning candidate could become a losing candidate had *more* voters ranked them higher in some ballots). I have published one article in this field ([9]), which was joint work with Nick Zayatz, an undergraduate at High Point University at the time of the research. That result found that the frequency of voting anomalies in the empirical data was generally lower than what theoretical researchers had projected. This research is ongoing, as more municipalities and states (such as Maine) begin to use RCV, and as I mine the data both to reinforce previous conclusions and search for other anomalies.

I also do theoretical research related to RCV, some of which overlaps with the applied research. For example, Lepelley et al ([11]) found that certain necessary and sufficient conditions need to be present for a three-candidate monotonicity anomaly to occur. I work on generalizing those necessary and sufficient conditions to higher numbers of candidates, as well as investigating how those conditions relate to other anomalies such as No-show violations. This work is of interest as intrinsic research, and it also helps to speed up computer programs I use to mine empirical data for various anomalies. One of my undergraduate research projects was connected to these theoretical aspects of ranked-choice voting.

Gerrymandering

Gerrymandering is a more recent addition to my research portfolio, and while I have not yet published in this area, I am engaged in active applied and theoretical research in gerrymandering. Most of my work in this area has been connected to the Metric Gerrymandering work initiated by Dr. Moon Duchin at Tufts University (`https://mggg.org/`), and specifically associated with the work of the Duke gerrymandering group (`https://sites.duke.edu/quantifyinggerrymandering/`).

A subset of us in the Duke group are working on an outlier analysis of Arizona, specifically the redistricting plan created in 2011 by their independent commission. Many papers (for example: [3], [4], [10]) have analyzed gerrymandered districts created by state legislatures, and our goal is to apply those same analyses to the independent commission to see if its districts are *less* gerrymandered. This project involves procuring and cleaning large sets of voting data, as well as using a computer program which employs a Markov Chain Monte Carlo (MCMC) method to create a large ensemble of viable districtings for a given state. This research is ongoing and involves professors and an undergraduate from 4 different institutions. I have also done undergraduate research investigating whether ranked-choice voting would reduce the efficiency gap (a metric used to measure gerrrymandering).

One of the challenges of using the MCMC method to create an ensemble of sample districtings is that it is not immediately clear if the ensemble is representative of all possible districtings for a given state. One goal is to get a sense of the geometry of the space of all possible redistrictings, and use that knowledge to help understand if the ensemble is representative. This interest in the geometry of the space led to my theoretical research in gerrymandering, specifically investigating different distance metrics in the space of redistricting. This has been investigated by an undergraduate student (who was working with my colleague Dr. Laurie Zack) and I am also actively pursuing this research with my colleague Dr. Edward Fuselier, with whom I have an article in preparation.

Recreational Mathematics

Recreational Mathematics is a broad term, and some of my research falls within this field. Most recently I have worked to analyze a dynamical system that arises from a child's game called Enemy-Protector. We created a mathematical model to investigate the potential end states that can arise from various initial conditions such as location of players and choice of Enemy/Protector. This is also joint work with Dr. Edward Fuselier, and we also are preparing an article related to this work.

I have also investigated the game Mat-Rix-Toe, a linear algebra game related to Tic-Tac-Toe, in which players place 1s and 0s into a matrix, trying to make the matrix either invertible or noninvertible. I created a linear algebra project related to the game, and presented on the results. Dr. Elin Farney and Dr. Julianna Stockton also used the project in their linear algebra classes, and we wrote an article about our experiences ([8]). I also did undergraduate research investigating which player would be the winner of a game of Mat-Rix-Toe under a variety of different conditions.

My most recently completed undergraduate research project involved the following seemingly simple scenario: Suppose a professor promised to drop the lowest homework score for students at the end of the semester. If all assignments are out of the the same score (that is, equally weighted), then the decision is easy–you simply drop the score that has the lowest percentage. However, if assignments are weighted differently, the assignment with the lowest percentage might not be the same as the assignment with the largest number of lost points, and it is not obvious which grade should be dropped. With my student researcher, we investigated how to decide which grade to drop, and discovered along the way that Blackboard, the common Course Management System, does not actually drop the optimal assignment if you use its feature to drop the lowest grade.

Scholarship of Teaching and Learning (SoTL)

I have done a number of SoTL projects over the past 10 years, three of which have resulted in publications. The aforementioned Mat-Rix-Toe project, which resulted in a publication, is one example ([8]). I also joined with colleagues to investigate the benefits of the flipped method of teaching–we each taught one section of a course in a traditional format and another flipped, and then compared quantitative and qualitative results. That project received internal funding from High Point University, and it resulted in a publication ([12]). A similar project with colleagues investigated the benefits and drawbacks of teaching Calculus 1 with a mixed group of students (both those who have taken AP Calculus and those who have not) as compared to separate classes for those two different populations. That research also resulted in a publication ([2]). I have also done projects that did not result in publication. For example, with my colleague Dr. Jenny Fuselier, we investigated the impact of a metacognitive intervention on students who had done poorly on their first test in a course. Due to some flaws in the data and data collection, we did not feel the work was high enough quality for publication, although we did present on the work at a regional conference.

Attitude toward Research

While many of my research interests align with "standard" research areas of mathematics, over the last 10 years I have shifted away from algebra, which was the research area of my dissertation and publications on the Mellin and Fourier transforms ([5], [6]). Recently, I have spent time applying math to popular culture in not only political science and recreational mathematics but also applying mathematical ideas, such as impossibility theorems, to topics such as the naming of children ([7]). I particularly enjoy working with undergraduate students on research projects in which they can start with something of interest, and then work on ways to investigate it mathematically. While this mostly occurs in my undergraduate research projects, I have also implemented these math research strategies in courses I have developed, including projects in my Game Theory, Math of Democracy, and Exploration in Mathematical Research courses. In this way, I have attempted to incorporate math research into my work as much as possible, wherever it can fit.

References

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