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Quiz 4C, Precalculus-04

Dr. Graham-Squire, Fall 2013

Name: Key

1. (2 points) Use the definition of logarithm to rewrite the equation into exponential form and then solve it.

$$\log_2 8 = x$$

$\Rightarrow 2^x = 8$ $\qquad 2^{?} = 8$

$\Rightarrow \boxed{x = 3}$

2. (4 points) Use laws of logarithms to expand the expression as much as possible:

$$\ln\left(\frac{x^2}{y^4\sqrt{x+2}}\right)$$

$= \ln x^2 - \ln\left(y^4\sqrt{x+2}\right)$

$= 2\ln x - \ln y^4 - \ln (x+2)^{1/2}$

$= 2\ln x - 4\ln y - \frac{1}{2}\ln(x+2)$

3. (4 points) An infectious disease is modeled by the equation

$$v(t) = \frac{4,000}{5 + 1995e^{-0.97t}}$$

where t is in days and $v(t)$ is the number of people who are infected with the disease.

(a) How many infected people are there initially?

(b) How many people will become infected as t goes to infinity?

(c) Suppose you wanted to know on what day the 40th person is infected. Explain how you would do it (you would need a calculator to get the exact answer, but you should be able to find an expression that represents the answer).

$e^0 = 1$

(a) at $t=0$ $\quad v(0) = \frac{4000}{5+1995e^{-0.97(0)}} = \frac{4000}{5+1995e^0}$

$= \frac{4000}{2000} = \boxed{2 \text{ people}}$

(b) as $t \to \infty$, $e^{-0.97t} \to e^{-\infty} = \frac{1}{e^{\infty}} \to 0$

So as $t \to \infty$, $v(t) \to \frac{4000}{5+1995(0)} = \frac{4000}{5} = \boxed{800}$

(c) $40 = \frac{4000}{5+1995e^{-0.97t}}$ now solve for t

$40(5+1995e^{-0.97t}) = 4000$

$200 + 40(1995e^{-0.97t}) = 4000$

$\frac{(40)(1995)e^{-0.97t}}{40(1995)} = \frac{3800}{40(1995)}$

$e^{-0.97t} = \frac{3800}{40(1995)}$

$\Rightarrow \boxed{t = \frac{\ln\left(\frac{3800}{40(1995)}\right)}{-0.97}} = \frac{-0.97t}{-0.97}$

Quiz 4D, Precalculus-04

Dr. Graham-Squire, Fall 2013

Name: Key

1. (2 points) Use the definition of logarithm to rewrite the equation into exponential form and then solve it.

$$\log_3 9 = x$$

$\Rightarrow 3^x = 9$

$\boxed{x = 2}$

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2. (4 points) Use laws of logarithms to expand the expression as much as possible:

$$\ln\left(\frac{a^3}{b^2\sqrt{a+4}}\right)$$

$= \ln a^3 - \left(\ln b^2\sqrt{a+4}\right)$ ✓

$= 3\ln a - \left(\ln b^2 + \ln\sqrt{a+4}\right)$ ✓

$= 3\ln a - 2\ln b - \frac{1}{2}\ln(a+4)$ ✓✓

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3. (4 points) An infectious disease is modeled by the equation

$$v(t) = \frac{6,000}{5 + 1995e^{-0.97t}}$$

where t is in days and $v(t)$ is the number of people who are infected with the disease.

(a) How many infected people are there initially?

(b) How many people will become infected as t goes to infinity?

(c) Suppose you wanted to know on what day the 50th person is infected. Explain how you would do it (you would need a calculator to get the exact answer, but you should be able to find an expression that represents the answer).

1.5 (a) "initially" means @ $t=0$ $\qquad e^0 = 1$

$$V(0) = \frac{6000}{5+1995e^{-0.97(0)}} = \frac{6000}{5+1995e^0} = \frac{6000}{2000} = \boxed{3} \text{ people}$$

1.5 (b) as $t \to \infty$, $e^{-0.97t} \to e^{-\infty} = \frac{1}{e^{\infty}} = 0$

So as $t \to \infty$, $v(t) \to \frac{6000}{5+1995(0)} = \frac{6000}{5} = \boxed{1200 \text{ people}}$

1 (c) Set $50 = \frac{6000}{5+1995e^{-0.97t}}$ and solve for t

$$\Rightarrow 50(5+1995e^{-0.97t}) = 6000$$

$$250 + 50(1995)e^{-0.97t} = 6000$$

$$-250 \qquad\qquad -250$$

$$\frac{50(1995)e^{-0.97t}}{50(1995)} = \frac{5750}{50(1995)}$$

$$e^{-0.97t} = \frac{5750}{50(1995)}$$

$$\Rightarrow \frac{\ln\left(\frac{5750}{50(1995)}\right)}{-0.97} = \frac{-0.97t}{-0.97}$$

$$t = \frac{\ln\left(\frac{5750}{50(1995)}\right)}{-0.97}$$