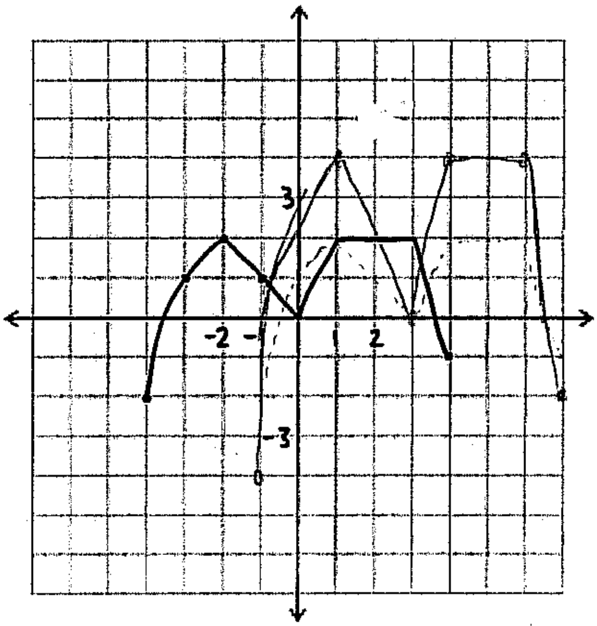
Quiz 2A, Precalculus

Dr. Graham-Squire, Fall 2013

12:10
12:14
4 ⇒ 15→20 min.

Name: Key

1. (2 points) A graph of $f(x)$ is given. Sketch the graph for $2f(x-3)$ on the same set of axes.



2. (3 points) Let $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = x^2 - 4x$. Find the composite functions and simplify your answer if possible.

(a) $(f \circ g)(x) = f(g(x)) = \frac{1}{\sqrt{g(x)}} = \boxed{\frac{1}{\sqrt{x^2-4x}}}$

(b) $g(g(x))$

$= g(x^2-4x)$

$= (x^2-4x)^2 - 4(x^2-4x)$

$= x^4 - 8x^3 + 16x^2 - 4x^2 + 16x$

$= x^4 - 8x^3 + 12x^2 + 16x$

3. (3 points) Find the inverse of the function $f(x) = \dfrac{4}{\sqrt{x+7}}$

$$y = \frac{4}{\sqrt{x+7}} \quad \checkmark$$

$$x = \frac{4}{\sqrt{y+7}} \quad \checkmark$$

$$x^2 = \frac{4^2}{y+7} \quad \checkmark$$

$$(y+7)(x^2) = 16 \quad \checkmark$$

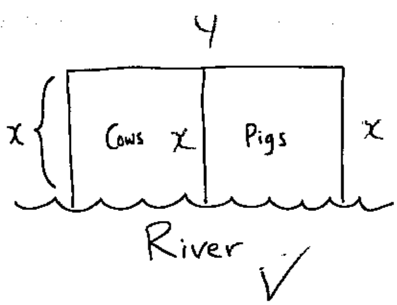
$$y+7 = \frac{16}{x^2}$$

$\checkmark$

$$\boxed{y = \frac{16}{x^2} - 7 = f^{-1}(x)}$$

$\checkmark$

4. (2 points) A farmer has 270 feet of fencing, and wants to build a rectangular pen for his cows and pigs. Part of the pen needs to be a fence down the middle so that the cows and pigs don't mix, and one side of the pen will be open to the river so the cows and pigs can drink. A picture is given below. Let x be the width of the pen (perpendicular to the river). Write a quadratic function (in terms of x) to represent the area of the pen. (1 extra credit point if you can tell me what the x-value is that gives the *maximum* area for the pen).



$\checkmark$

$$y + 3x = 270$$

$$y = 270 - 3x \quad \checkmark$$

$$\text{Area} = l \cdot w \quad \checkmark$$

$$A(x) = x(270 - 3x) \quad \checkmark$$

$$\boxed{A(x) = 270x - 3x^2}$$

Ex. cred: Max area is at $x = \frac{-b}{2a} = \frac{-270}{2(3)} = \frac{-90}{-2} = 45$

$$\boxed{x = 45}$$

Quiz 2B, Precalculus

Dr. Graham-Squire, Fall 2013

Name: Key

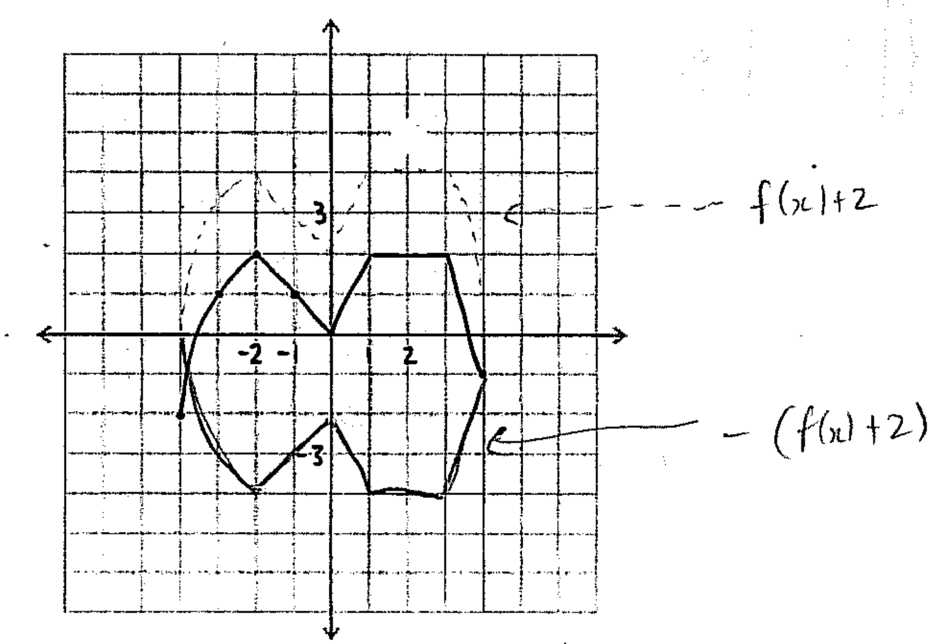
1. (3 points) Let $f(x) = \dfrac{1}{\sqrt{x}}$ and $g(x) = x^2 - 4x$. Find the composite functions and simplify your answer if possible.

(a) $(g \circ f)(x)$ $= g\left(\frac{1}{\sqrt{x}}\right) = \left(\frac{1}{\sqrt{x}}\right)^2 - 4\left(\frac{1}{\sqrt{x}}\right) = \boxed{\frac{1}{x} - \frac{4}{\sqrt{x}}}$

(b) $f(f(x))$

$f\left(\frac{1}{\sqrt{x}}\right) = \dfrac{1}{\sqrt{\left(\frac{1}{\sqrt{x}}\right)}} = \dfrac{1}{\left(\frac{\sqrt{1}}{\sqrt{\sqrt{x}}}\right)} = 1 \cdot \dfrac{\sqrt{\sqrt{x}}}{1} = \boxed{\sqrt{\sqrt{x}}}$ or $x^{1/4}$

2. (2 points) A graph of $f(x)$ is given. Sketch the graph for $-(f(x)+2)$ on the same set of axes.



3. (3 points) Find the inverse of the function $f(x) = \dfrac{5}{\sqrt{x-1}}$

$$y = \frac{5}{\sqrt{x-1}}$$

$$\frac{x}{1} = \frac{5}{\sqrt{y-1}}$$

$$5 = x\sqrt{y-1}$$

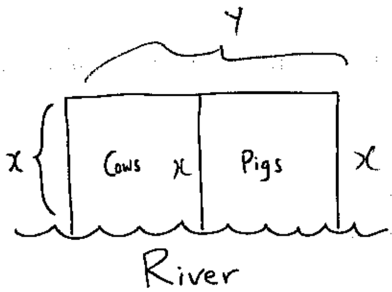
$$\left(\frac{5}{x}\right)^2 = \left(\sqrt{y-1}\right)^2$$

$$\frac{25}{x^2} = y - 1$$

$$\rightarrow y = \frac{25}{x^2} + 1$$

$$\boxed{f^{-1}(x) = \frac{25}{x^2} + 1}$$

4. (2 points) A farmer has 180 feet of fencing, and wants to build a rectangular pen for his cows and pigs. Part of the pen needs to be a fence down the middle so that the cows and pigs don't mix, and one side of the pen will be open to the river so the cows and pigs can drink. A picture is given below. Let x be the width of the pen (perpendicular to the river). Write a quadratic function (in terms of x) to represent the area of the pen. (1 extra credit point if you can tell me what the x-value is that gives the *maximum* area for the pen).



$$3x + y = 180$$

$$y = 180 - 3x$$

$$\text{Area} = l \cdot w = x \cdot y$$

$$\text{Area} = (180 - 3x)\,x$$

$$\boxed{A(x) = 180x - 3x^2}$$

Ex Cred.

$$\text{Max } @ \ \frac{-b}{2a} = \frac{-180}{2(-3)} = \frac{-180}{-6} = \boxed{30}$$