Test 1 - MTH 1400 Online

Dr. Graham-Squire, Summer 2016

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 7 questions of the test, however you should still show all of your work. No calculators are allowed on the last 5 questions. If you change from the With calculator portion of the test to the No Calculator portion, it is fine to go back to the With Calculator portion again. However, once you turn in the No Calculator portion of the test, you CANNOT return to it.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 12. Total Points = 70.

1. (10 points) A farmer wants to build a rectangular pen in which to keep some of her animals. She needs to keep the animals separate, though, so she decides to put one line of fencing lengthwise and two lines of fencing widthwise–with the fencing around the outside, this will create 6 rectangular pens enclosed by the fence.

2 (a) Draw a diagram of the situation. Let x be the length of the pen and y be the width.

2 (b) If the farmer wants the pen to be x = 40 feet long and y =15 feet wide on the outside (so each individual pen would be smaller than that), how much fencing does she need total? Note that you need to include the fencing that goes across the inside as well as the fencing on the outside.

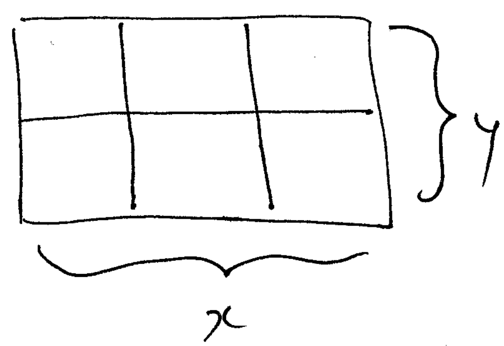
2 (c) Write a general expression for the total amount of fencing needed in terms of the variables x and y.

2 (d) If the farmer had 1200 total feet of fencing, and she wanted the pen to be 250 feet long on the outside, how wide could she make the pen? (Note: it might help to use your answer from (c))

✓ (e) Assuming that the farmer has 1200 feet of fencing, the general expression for the total *area* of the pen in terms of the variable x is given by $A(x) = 300x - \frac{3}{4}x^2$. Explain why that equation makes sense (that is, explain where it comes from).

✓ (f) Use the equation from (e) to find the dimensions of the pen that would give a *maximum* area. Show your work (or explain how you came up with your answer) and explain how you know it is a maximum.

(a)



(b) $40+40+40+4(15) = 120+60 = 180$ feet of fencing.

(c) Fencing $= 3x + 4y$

(d) $1200 = 3(250) + 4y \Rightarrow 1200 = 750 + 4y$

$-750 \quad -750$

$\frac{450}{4} = \frac{4y}{4}$

$\boxed{y = 112.5 \text{ ft}}$

Use this extra space to answer question 1.

(e) $1200 = 3x + 4y$

$\frac{1200}{4} - \frac{3x}{4} = \frac{4y}{4}$

$300 - \frac{3}{4}x = y$

Area $= xy$

$A(x) = x\left(300 - \frac{3}{4}x\right)$

$\boxed{A(x) = 300x - \frac{3}{4}x^2}$

Solve (c) equation for y, then plug into $A(x) = xy$.

(f) $A(x)$ will give max b/c $-\frac{3}{4}$ is negative (that is, leading coeff is neg). Max will occur at

$x = \frac{-b}{2a} = \frac{-300}{2\left(-\frac{3}{4}\right)} = -300\left(-\frac{2}{3}\right) = 200$

and $y = 300 - \frac{3}{4}(200) = 300 - 3(50) = 300 - 150 = 150$

$\Rightarrow$ dimensions are 200 ft long, 150 feet wide

2. (5 points) The table gives the position of a particle moving in a straight line at certain times, where t is time in seconds and p is the position in feet.

t	p
1.5	7.25
1.9	16.777
1.95	18.344
1.99	19.662
2	20

Calculate the average rate of change of the particle from

(a) 1.5 to 2 seconds

(b) 1.9 to 2 seconds

(c) 1.95 to 2 seconds

(d) 1.99 to 2 seconds

(e) Use your answers from (a) through (d) to estimate how fast (in feet/sec) the particle was moving at exactly 2 seconds.

(a) $\dfrac{f(2)-f(1.5)}{2-1.5} = \dfrac{20-7.25}{0.5} = 25.5$ ft/sec.

(b) $\dfrac{f(2)-f(1.9)}{2-1.9} = \dfrac{20-16.777}{0.1} = 32.23$ ft/sec

(c) $\dfrac{20-18.344}{0.05} = 33.12$ ft/sec

(d) $\dfrac{20-19.662}{0.01} = 33.8$ ft/sec

(e) It looks like it is approaching $\boxed{34 \text{ ft/sec.}}$ at 2 sec.

3. (5 points) Suppose that Dominic's Lego collection increases in a linear fashion. At age 5 he had 586 Legos, and at age 10 he had 2051 Legos.

(a) Write an equation of a line that models the growth of Dominic's Lego collection. Let x be Dominic's age and y be the number of Legos he owns.

(b) How old was Dominic when he received his first Legos?

(a) $(5, 586)$ $(10, 2051)$

$$\text{slope} = \frac{2051 - 586}{10 - 5} = 293$$

$y = mx + b$

$586 = 293(5) + b$

$586 = 1465 + b$

$-879 = b$

$\Rightarrow \boxed{y = 293x - 879}$

(b) Need to know when $y = 0$

$0 = 293x - 879$

$$\frac{879}{293} = \frac{293x}{293}$$

$3 = x$

He was 3 years old.

4. (5 points) Let $f(x) = \frac{1}{x}$ and $g(x) = 2x + 3$.

1.5 (a) Calculate the composition of functions $(f \circ g)(x)$.

1.5 (b) Calculate the composition of functions $(g \circ f)(x)$.

1.5 (c) Calculate the *inverse* function of $(f \circ g)(x)$ (that is, find $(f \circ g)^{-1}(x)$). (d) Is $(f \circ g)^{-1}(x) = (g \circ f)(x)$? 0.5

(a) $f(g(x)) = \frac{1}{(2x+3)}$

(b) $g(f(x)) = 2\left(\frac{1}{x}\right) + 3 = \frac{2}{x} + 3$

(c) $y = \frac{1}{2x+3} \longrightarrow x = \frac{1}{2y+3}$

$\Rightarrow (2y+3)x = 1$

$2yx + 3x = 1$

$\frac{2yx}{2x} = \frac{1-3x}{2x}$

$y = \frac{1-3x}{2x}$

or

$y = \frac{1}{2x} - \frac{3}{2}$

(d) <u>No</u> $\frac{2}{x} + 3 \neq \frac{1}{2x} - \frac{3}{2}$

5. (10 points) The goal of this problem is to fully factor the polynomial $f(x) = x^5 + x^4 + 5x^3 + 5x^2 - 36x - 36$ by doing the following:

4 (a) Use polynomial long division to calculate $(x^5+x^4+5x^3+5x^2-36x-36) \div (x^2-4)$. You should have no remainder.

4 (b) Use your result from (a), and possibly other factoring techniques, to fully factor $f(x)$. You may need to use the quadratic formula.

2 (c) What are the real zeroes of $f(x)$? What are the complex zeroes of $f(x)$?

(a)
$$
\begin{array}{r l}
 & x^3 + x^2 + 9x + 9 \\
x^2-4 \,\big) & x^5 + x^4 + 5x^3 + 5x^2 - 36x - 36 \\
 & -(x^5 \quad - 4x^3) \\
 & x^4 + 9x^3 + 5x^2 \\
 & -(x^4 + 0 - 4x^2) \\
 & 9x^3 + 9x^2 - 36x \\
 & 9x^3 + 0 - 36x \\
 & 9x^2 + 0x - 36 \\
 & 9x^2 \quad - 36 \\
 & 0
\end{array}
$$

(b) $(x^2-4)(x^3+x^2+9x+9) = f(x)$

$f(x) = (x-2)(x+2)\left(x^2(x+1) + 9(x+1)\right)$

$= \cancel{(x-2)(x+2)(x+1)(x^2+9)}$

$\boxed{f(x) = (x-2)(x+2)(x+1)(x+3i)(x-3i)}$

-1 for no $(x^2-4) = (x+2)(x-2)$

-0.5 for incorrect factoring of x^2+9

(c) real zeroes are $2, -2, -1$

complex zeroes are $-3i, 3i$

6. (5 points) A sky diver jumps out of a plane and her downward velocity is given by

$$v(x) = 160(1 - e^{-0.2x})$$

where x is measured in seconds and $v(x)$ is measured in feet per second.

2 (a) Find the initial velocity of the sky diver.

2 (b) How fast is the sky diver moving after 6 seconds? Round to nearest 0.01

1 (c) The *terminal velocity* of a sky diver is the maximum speed she reaches as time goes on. What is the terminal velocity for the sky diver in this situation, and how did you calculate it?

(a) $v(0) = 160(1-e^{-0.2(0)}) = 160(1-e^{0}) = 160(1-1) = \boxed{0}$

(b) $v(6) = 160(1-e^{-0.2(6)}) = \boxed{111.81 \text{ ft/sec.}}$

18 : 19

(c) What happens as $x \to \infty$?

$$e^{-0.2x} = \frac{1}{e^{0.2x}} \to \frac{1}{\infty} \quad \text{as } x \to \infty$$

and $\frac{1}{\infty} = 0 \Rightarrow$

terminal velocity $= 160(1-0) = 160(1) = \boxed{160 \text{ feet/sec.}}$

7. (5 points) Use logarithm laws to fully expand the expression $\ln\sqrt{\frac{x^3}{x^2-7}}$.

$$\ln\sqrt{\frac{x^3}{x^2-7}} = \ln\left(\frac{x^3}{x^2-7}\right)^{1/2}$$

$$= \frac{1}{2}\ln\left(\frac{x^3}{x^2-7}\right)$$

$$= \frac{1}{2}\left(\ln x^3 - \ln(x^2-7)\right)$$

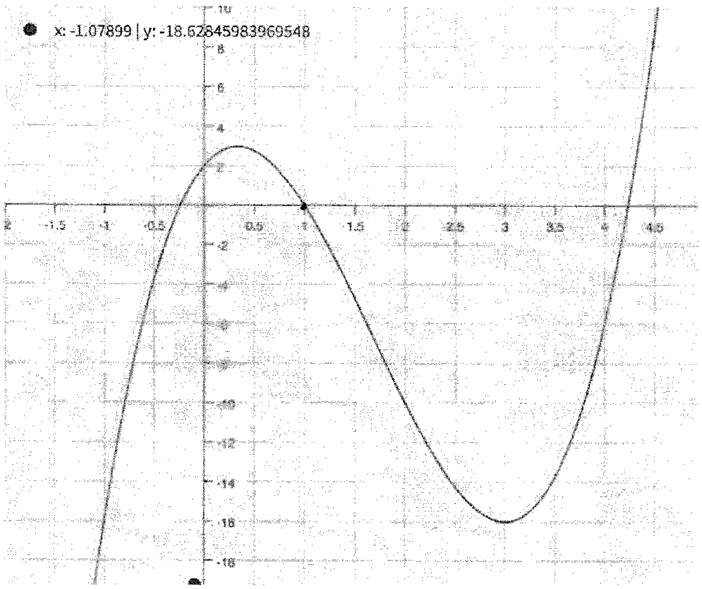
$$= \boxed{\frac{1}{2}\left(3\ln x - \ln(x^2-7)\right)}$$

1 for work.

or

$$\boxed{\frac{3}{2}\ln x - \frac{1}{2}\ln(x^2-7)}$$

Extra Credit(2 points) Below is the graph of a polynomial $f(x)$. The number $2+\sqrt{5}$ is one of the zeroes of $f(x)$. Use that fact and information from the graph to find an expression for $f(x)$.



$x = 1$ is a zero $\Rightarrow f(x) \overset{?}{=} (x-1)(x-(2+\sqrt{5}))(x-(2-\sqrt{5}))$

$= (x-1)(x^2-(2+\sqrt{5}+2-\sqrt{5})x+(4-5))$

$= (x-1)(x^2-4x-1)$

$f(x) \overset{?}{=} x^3-5x^2+3x+1$

<u>but</u> has y-int. of 1, and need y-int. of 2

$\Rightarrow f(x) = 2(x^3-5x^2+3x+1)$

No Calculator Name: *Key*

- Note that once you finish this portion of the test and turn it in, you CANNOT return to it!

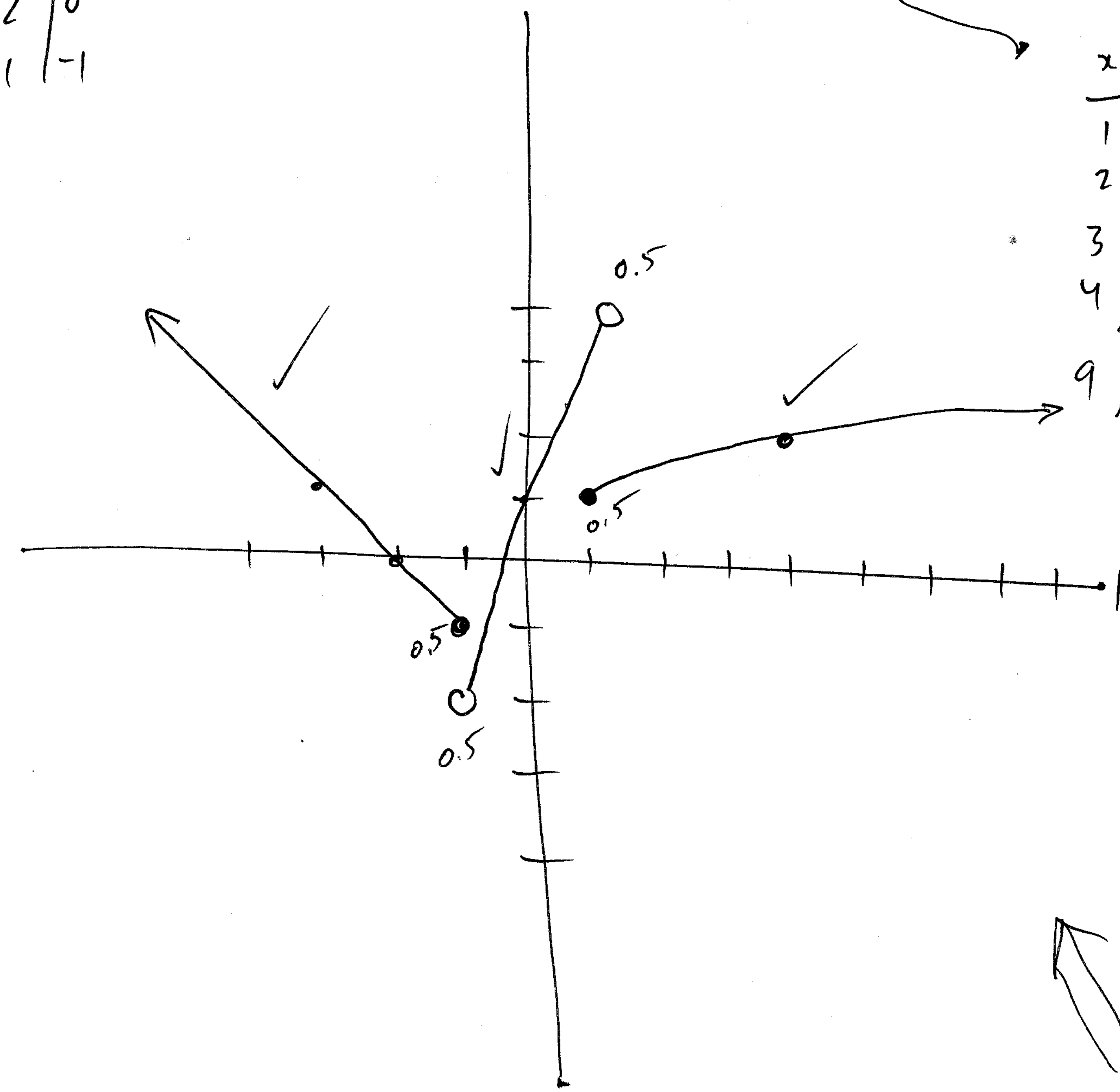
- If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

8. (5 points) Graph the piecewise function $f(x) = \begin{cases} |x| - 2 & \text{if } x \le -1 \\ 3x + 1 & \text{if } -1 < x < 1 \\ \sqrt{x} & \text{if } x \ge 1 \end{cases}$.

x	y
-1	-2
0	1
1	4

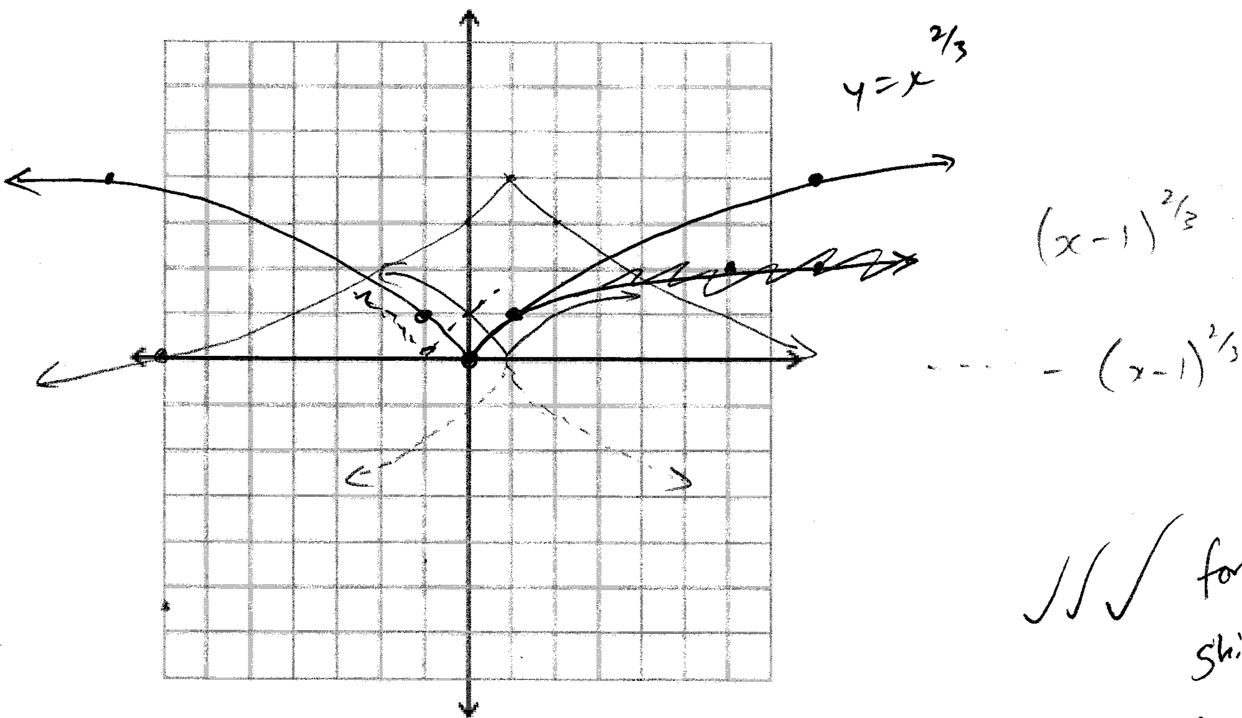
x	y
-3	1
-2	0
-1	-1

x	y
1	1
2	
3	
4	2
9	3



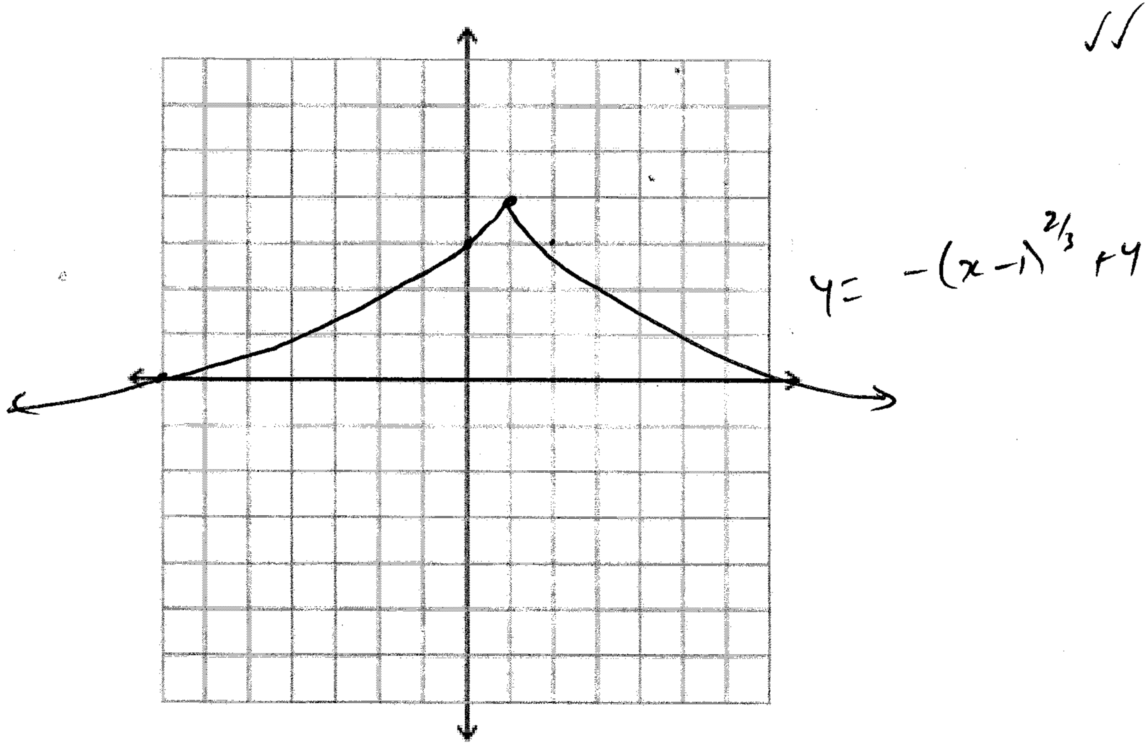
9. (5 points) You are given the graph of $y = x^{2/3}$. Use your knowledge of transformation of functions to sketch the graph of $y = -(x-1)^{2/3} + 4$. You can sketch the new graph on the axes with the original or on the blank graph below it.

NEED TO SKETCH GRAPH BY HAND AND INSERT

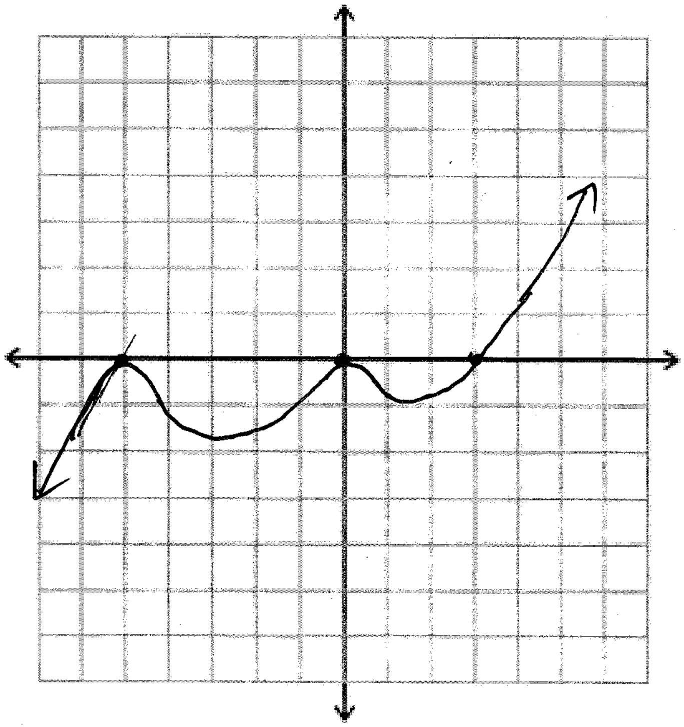


✓✓✓ for each shift

✓✓ for work shown



10. (5 points) Sketch a graph of the polynomial $y = x^2(x-3)^3(x+5)^4$. Below your graph, explain in words (a) how you knew where to put the x-intercepts, (b) how you knew where the graph was positive or negative, and (c) how you knew what the end behavior was.



$y = x^2 (x-3)^3 (x+5)^4$

zero = 0 ↓ mult = 2 ⇒ bounce

3 ↓ mult = 3 ⇒ pass through

−5 ↘ bounce

check $x = -6$ $+ \cdot - \cdot + \Rightarrow -$

$x = -4$ $+ \cdot - \cdot + \Rightarrow -$

$x = 1$ $+ \cdot - \cdot + \Rightarrow -$

$x = 4$ $+ \cdot + \cdot + \Rightarrow +$

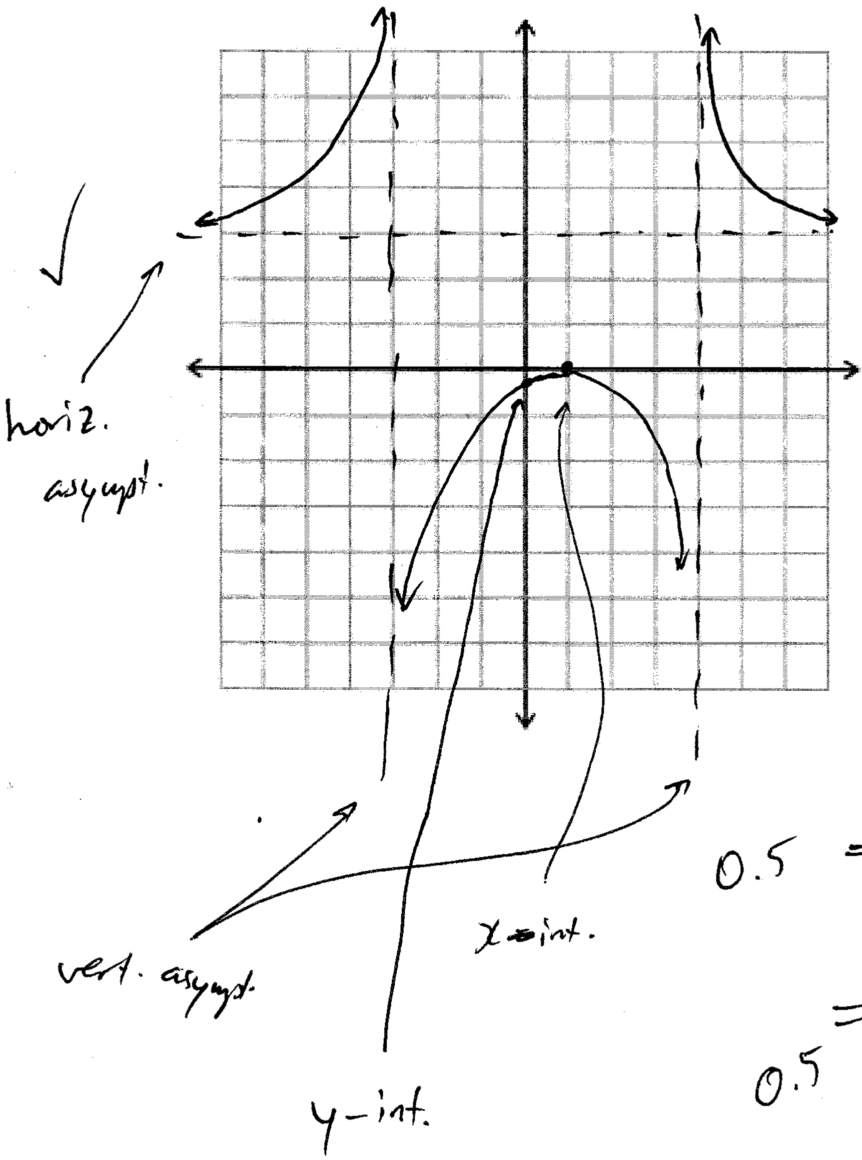
(a) x-intercepts at $\left.\begin{array}{l} x^2 = 0 \\ (x-3) = 0 \\ (x+5) = 0 \end{array}\right\} \rightarrow$ That is, the zeroes.

(b) To find pos/neg, I did test points

(c) leading term is x^9 ⇒ as $x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to -\infty$.

11. (5 points) Sketch a graph of the rational function $f(x) = \dfrac{3x^2 - 6x + 3}{x^2 - x - 12}$. On your graph you should label the following, if they exist: x-intercept(s), y-intercept(s), horizontal asymptote(s) and vertical asymptote(s.)



$$f(x) = \frac{3(x^2-2x+1)}{(x-4)(x+3)}$$

mult = 2 $\Rightarrow$ bounces

$$= \frac{3(x-1)(x-1)}{(x-4)(x+3)}$$

$\Rightarrow$ x int. at $x=1$ b/c $x-1=0$ 0.5

$\Rightarrow$ vert asympt. at $x=4$ and $x=-3$ 0.5

0.5 $\Rightarrow$ y-int. at $f(0) = \frac{+3}{-12} = \frac{-1}{4}$

0.5 $\Rightarrow$ horiz. asympt. of $y=3$ b/c as $x \to \infty, -\infty$, have $\frac{3x^2}{x^2} = 3$.

12. (5 points) Solve the equation $\log_4 2 + \log_4 \frac{x}{2} = 3$ by doing the following:

(a) Use logarithm laws to combine the left-hand side of the equation into a single logarithm.

(b) Rewrite the logarithmic equation into exponential form and solving.

(a) $\log_4 2 + \log_4 \left(\frac{x}{2}\right) = 3$ ✓

$\Rightarrow \log_4 \left(2 \cdot \frac{x}{2}\right) = 3$ ✓ ✓

$\Rightarrow \log_4 x = 3$

Give 3 for $\log_4 \left(2 \cdot \frac{x}{2}\right) = 3$

(b) $x = 4^3 \rightarrow \boxed{x = 64}$ ✓ ✓

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