

Test 3A, Math 130.001

SSII, 2009

Name: Key

PID Number: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1. Show all of your work. A correct answer with insufficient work will be counted wrong.
2. Clearly indicate your answer by putting a box around it.
3. Calculators are allowed on this exam, but NOT cell phones or laptops.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Total number of questions = ¹⁰10. Total points = 100.
6. Make sure you sign the pledge and write your PID on both pages.

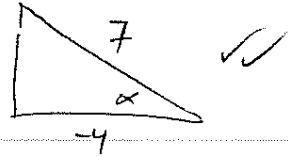
⇒ Q II

1. (10 points) Let $\cos \alpha = -\frac{4}{7}$ and $\tan \alpha < 0$. Evaluate the following:

(a) $\cos\left(\alpha - \frac{\pi}{4}\right) = \cos \alpha \cos \frac{\pi}{4} + \sin \alpha \sin \frac{\pi}{4}$

$= \frac{-4}{7} \cdot \frac{\sqrt{2}}{2} + \frac{+\sqrt{33}}{7} \cdot \frac{\sqrt{2}}{2}$

$= \frac{-4\sqrt{2} + \sqrt{66}}{14}$ or 0.176



⇒ $\cos \alpha = -\frac{4}{7}$

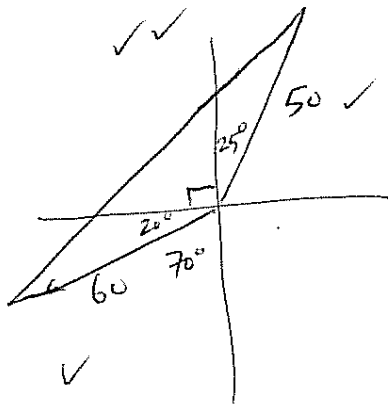
$\sin \alpha = \frac{+\sqrt{33}}{7}$

(b) $\csc(2\alpha) = \frac{1}{\sin(2\alpha)} = \frac{1}{2 \sin \alpha \cos \alpha} = \frac{1}{2 \left(\frac{+\sqrt{33}}{7}\right) \left(-\frac{4}{7}\right)}$

$= \frac{-49}{8\sqrt{33}}$ or $\frac{49\sqrt{33}}{264}$

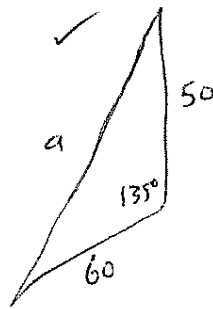
$= -1.066$

2. (10 points) At 2:00 PM a ship leaves port and travels N 25° E at a rate of 20 miles per hour. At 2:30 PM another ship leaves the same port and travels S 70° W at 30 miles per hour. To the nearest mile, how far apart are the ships at 4:30 PM?



$20 \cdot 2.5 = 50$

$30 \cdot 2 = 60$



SAS ⇒ Law of Cos. ✓

$a^2 = 50^2 + 60^2 - 2(50)(60)\cos 135^\circ$

$a^2 = 10342.64$

$a = 101.7$

$a = 102$ miles

1/2 point off for rounding.

3. (10 points) Approximate (to the nearest tenth of a degree) all solutions to the equation $3\sec^2\theta - 3\sec\theta - 1 = 0$ on the interval $[0^\circ, 360^\circ)$. *doesn't factor!*

$$\sec\theta = \frac{3 \pm \sqrt{9 - 4(3)(-1)}}{6} \quad \checkmark\checkmark$$

$$\sec\theta = \frac{3 \pm \sqrt{21}}{6} \quad \checkmark$$

$$\sec\theta = 1.26 \quad \text{or} \quad \sec\theta = -0.26 \quad \checkmark$$

$$\frac{1}{\cos\theta} = 1.26 \quad \checkmark$$

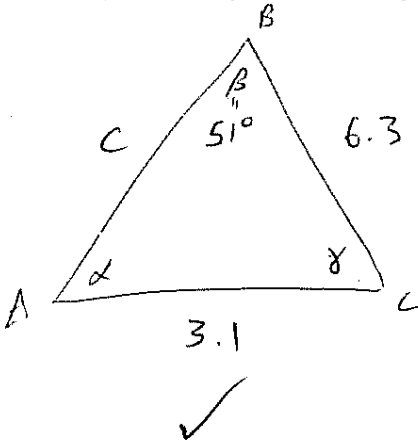
$$\cos\theta = \frac{1}{1.26}$$

$$\theta = \boxed{37.7^\circ} \quad \leftarrow \text{in QI} \quad \checkmark$$

$$\text{QIV} \Rightarrow \theta = 360 - 37.7 = \boxed{322.3^\circ} \quad \checkmark\checkmark$$

Not in the range of secant

4. (10 points) Solve triangle ABC given $a=6.3$, $b=3.1$, and $\beta=51^\circ$.



SSA, use law of sines. (or Law of cosines)

$$\frac{\sin 51^\circ}{3.1} = \frac{\sin \alpha}{6.3} \quad \checkmark\checkmark\checkmark\checkmark$$

$$\Rightarrow \sin \alpha = 1.579 \quad \checkmark$$

\Rightarrow no solution since $\checkmark\checkmark$

$\sin \alpha$ is never bigger than 1

5. (10 points) Find ALL solutions to the equation

$$\sin\left(3\theta - \frac{\pi}{6}\right) = \frac{1}{2}$$

Let $\alpha = 3\theta - \frac{\pi}{6} \Rightarrow \sin \alpha = \frac{1}{2}$ for $n \in \mathbb{Z}$

$$\alpha = \frac{\pi}{6} + 2\pi n \quad \text{or} \quad \alpha = \frac{5\pi}{6} + 2\pi n$$

$$3\theta - \frac{\pi}{6} = \frac{\pi}{6} + 2\pi n$$

$$\text{or } 3\theta - \frac{\pi}{6} = \frac{5\pi}{6} + 2\pi n$$

$$3\theta = \frac{\pi}{3} + 2\pi n$$

$$\text{or } 3\theta = \pi + 2\pi n$$

$$\theta = \frac{\pi}{9} + \frac{2\pi}{3}n$$

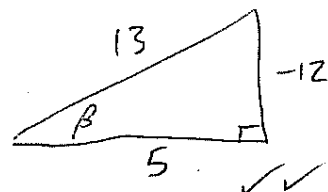
$$\text{or } \theta = \frac{\pi}{3} + \frac{2\pi}{3}n$$

6. (10 points) Let $\tan \beta = -\frac{12}{5}$ and $\sin \beta < 0$. Find the following:

\Rightarrow QIV

(a) $\sin(2\beta) = 2\sin \beta \cos \beta$

$$= 2 \left(-\frac{12}{13}\right) \left(\frac{5}{13}\right) = \frac{-120}{169}$$



$$\tan(2\beta) = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \left(-\frac{12}{5}\right)}{1 - \left(-\frac{12}{5}\right)^2} = \frac{-24}{5} = \frac{-24}{5} \cdot \frac{25}{-119} = \frac{120}{119}$$

(c) Which quadrant contains 2β ?

QIII

Since $\tan 2\beta > 0$ and $\sin 2\beta < 0$

7. (10 points) Find the exact solutions, on the interval $[0, 2\pi)$, to the equation

$$-\sin^2 \theta = \cos(2\theta)$$

$$0 = \cos 2\theta + \sin^2 \theta \quad \checkmark \checkmark$$

$$0 = 1 - 2\sin^2 \theta + \sin^2 \theta \quad \checkmark$$

$$\Rightarrow \sin^2 \theta = 1 \quad \checkmark$$

$$\sin \theta = \pm 1 \quad \checkmark \checkmark \checkmark$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad \checkmark \quad \text{or} \quad \frac{3\pi}{2} \quad \checkmark$$

8. (10 points) Find the exact value of $\sin\left(-\frac{5\pi}{12}\right) = \sin\left(-\frac{\pi}{4} + -\frac{\pi}{6}\right)$

$$\checkmark \checkmark \checkmark \quad \sin\left(-\frac{\pi}{4} + -\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{6}\right) + \cos\left(-\frac{\pi}{4}\right)\sin\left(-\frac{\pi}{6}\right) \quad \checkmark \checkmark$$

$$= \frac{-\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \left(-\frac{1}{2}\right) \quad \checkmark$$

$$= \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}} \quad \checkmark$$

or equivalently

$$-\sin\left(\frac{5\pi}{12}\right) = -\sqrt{\frac{1 - \cos\frac{5\pi}{6}}{2}}$$

$$= \sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \boxed{\frac{-\sqrt{2} + \sqrt{3}}{2}}$$

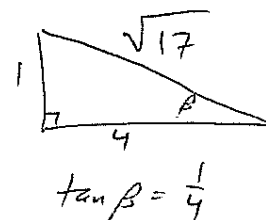
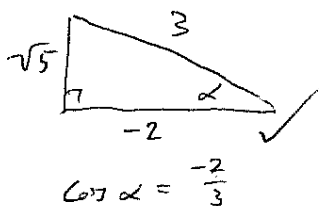
9. (10 points) Find the exact values of the following. Let $x > 0$.

(a) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \boxed{\frac{\pi}{6}}$

✓✓✓

(b) $\sin\left[\overbrace{\cos^{-1}\left(-\frac{2}{3}\right)}^{\alpha} + \overbrace{\tan^{-1}\left(\frac{1}{4}\right)}^{\beta}\right]$

✓✓✓



$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \frac{\sqrt{5}}{3} \cdot \frac{4}{\sqrt{17}} + \frac{-2}{3} \cdot \frac{1}{\sqrt{17}} \\ &= \boxed{\frac{4\sqrt{5} - 2}{3\sqrt{17}}} \end{aligned}$$

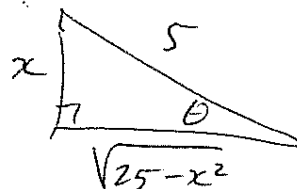
(c) $\cot\left(\arcsin\left(\frac{x}{5}\right)\right)$

Let $\theta = \arcsin\left(\frac{x}{5}\right)$

$\Rightarrow \sin \theta = \frac{x}{5}$

✓✓

$$\begin{aligned} &= \cot \theta \\ &= \boxed{\frac{\sqrt{25-x^2}}{x}} \end{aligned}$$



10. (10 points) Dominic and Zia are standing on opposite sides of the Golden Gate Bridge, looking down at a whale swimming under the bridge. If the angle of depression from Dominic to the whale is 68° , the angle of depression from Zia to the whale is 77° , and Dominic and Zia are 450 feet from each other, how high above the water is the bridge?

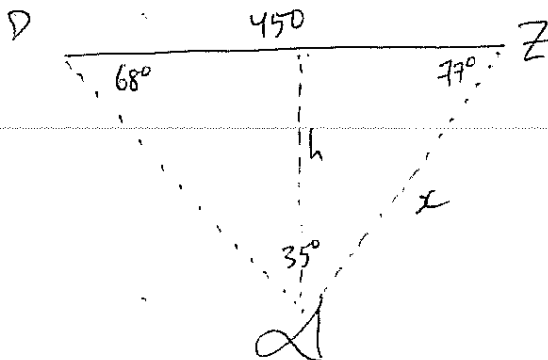
$$\frac{x}{\sin 68^\circ} = \frac{450}{\sin 35^\circ}$$

$$x = 727.423 \text{ feet}$$

$$\sin 77^\circ = \frac{h}{x}$$

$$h = \sin 77^\circ \cdot x$$

$$h = \boxed{709 \text{ feet}}$$



Extra Credit(2 points): Use an example to illustrate that $\sin(\alpha + \beta) \neq \sin \alpha + \sin \beta$

$$\sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right) \stackrel{?}{=} \sin \frac{\pi}{4} + \sin \frac{\pi}{4}$$

$$\sin\left(\frac{\pi}{2}\right)$$

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$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$\boxed{1 \neq \sqrt{2}}$$