Test 1A, Math 130.001

SSII, 2009

Name: Key

PID Number: ____________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

____
(*signature*)

DIRECTIONS

1. Show all of your work. A correct answer with insufficient work will be counted wrong.
2. Clearly indicate your answer by putting a box around it.
3. Calculators are allowed on this exam, but NOT cell phones or laptops.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Total number of questions = 10. Total points = 100.
6. Make sure you sign the pledge and write your PID on both pages.

PID Number: Key   Test A

1. (10 points) Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the function $f(x) = \sqrt{3x+5}$.

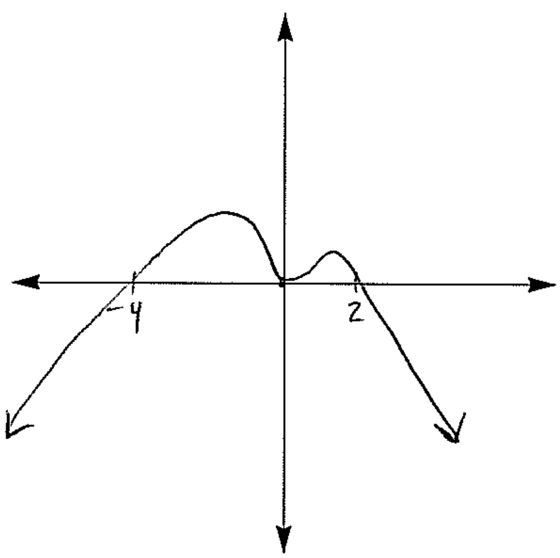
$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{3(x+h)+5} - \sqrt{3x+5}}{h} \cdot \frac{\sqrt{3(x+h)+5} + \sqrt{3x+5}}{\sqrt{3(x+h)+5} + \sqrt{3x+5}}$$

$$= \frac{\cancel{3x}+3h\cancel{+5} - (\cancel{3x}\cancel{+5})}{h\left(\sqrt{3(x+h)+5} + \sqrt{3x+5}\right)}$$

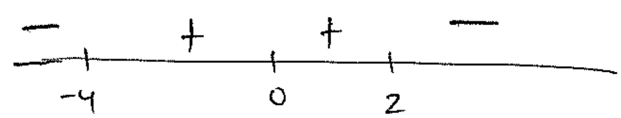
$$= \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3(x+h)+5} + \sqrt{3x+5}\right)}$$

$$= \boxed{\frac{3}{\sqrt{3(x+h)+5} + \sqrt{3x+5}}}$$

2. (10 points) Sketch the graph of $h(x) = -x^4 - 2x^3 + 8x^2$ (Note: Show your work! No credit will be given for a graph copied off a calculator with no supporting work.)



$$h(x) = -x^2(x^2 + 2x - 8)$$
$$= -x^2(x+4)(x-2)$$



(b) On what interval(s) is $h(x) < 0$? $h(x) < 0$ on $(-\infty, -4) \cup (2, \infty)$

3. (10 points) Write the equation of a rational function $g(x)$ that has vertical asymptotes at $x = a$ and $x = -3$, a hole at $x = -7$, an x-intercept at $x = 0$, and a horizontal asymptote at $y = 3$.

V.A. $\Rightarrow (x-a)(x+3)$ on bottom ✓✓

hole $\Rightarrow \frac{(x+7)}{(x+7)}$ ✓✓

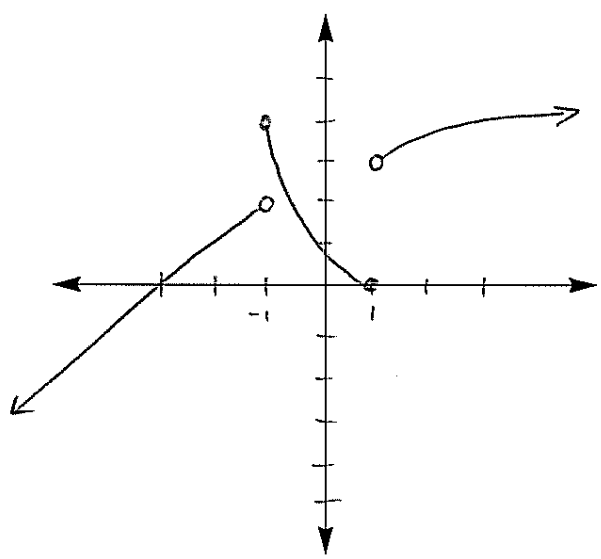
x-intercept $\Rightarrow x$ (to some power) on top ✓✓✓

H.A. at $y=3$ $\Rightarrow$ top and bottom have <u>same</u> degree and leading coefficient of top is 3. ✓

$$\Rightarrow \boxed{g(x) = \frac{3 \cdot x^2 \cdot (x+7)}{(x-a)(x+3)(x+7)}}$$ ✓

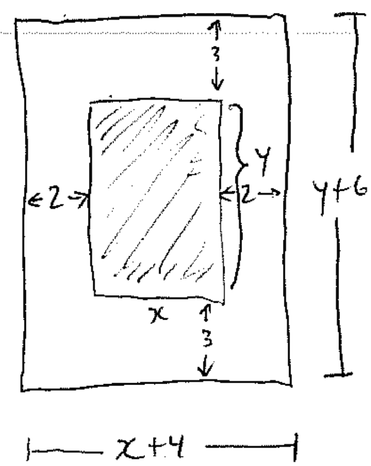
4. (8 points) Graph the following function:

$$f(x) = \begin{cases} x+3 & \text{if } x < -1 \\ (x-1)^2 & \text{if } -1 \le x \le 1 \\ \sqrt{x} + 2 & \text{if } x > 1 \end{cases}$$



5. (10 points) I am making a poster (see picture). The poster will be composed of a printed portion in the middle, with a white border around the outside. The printed portion (represented by the shaded area in the picture) must have an area of 30 in^2, but it can have any dimensions for its length and height. The top and bottom borders must be 3 inches wide and the left and right borders must be 2 inches wide. Write an expression for the total area of the poster in terms of x, the length of the bottom of the printed portion.

$$xy = 30 \text{ in}^2 \Rightarrow y = \frac{30}{x}$$



Total Area $= (x+4)(y+6)$

$$A(x) = (x+4)\left(\frac{30}{x} + 6\right)$$

$$= 30 + \frac{120}{x} + 6x + 24$$

$$\boxed{A(x) = 54 + 6x + \frac{120}{x}}$$

6. (12 points) Find the exact value of $\csc\theta$ and $\cos\theta$ if θ is in standard position and the point $(5, -3)$ in on its terminal side.

$$r = \sqrt{5^2 + (-3)^2}$$

$$r = \sqrt{25 + 9}$$

$$r = \sqrt{34}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{\sqrt{34}}{-3}$$

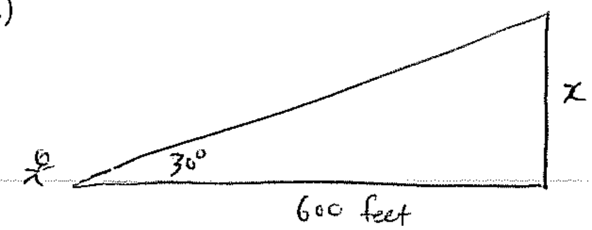
$$\boxed{\csc\theta = \frac{\sqrt{34}}{-3}}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{\sqrt{34}}$$

$$\boxed{\cos\theta = \frac{5}{\sqrt{34}}}$$

or $\boxed{\cos\theta = \frac{5\sqrt{34}}{34}}$

7. (10 points) A forester, 600 feet from the base of a tree, observes that the angle between the ground and the top of the tree is 30 degrees. Use trigonometry to find the height of the tree. (You should give the exact answer in simplified form. Two points will be taken off for a decimal approximation.)



−5 if they don't have tan.

$$\tan 30^\circ = \frac{x}{600 \text{ feet}} \qquad \tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \cdot 600 \text{ feet} = x$$

$$\boxed{\frac{600}{\sqrt{3}} \text{ feet} = x} \quad \text{or}$$

$$\frac{\cancel{600}^{200}\sqrt{3}}{\cancel{3}} \text{ feet} = x \qquad \boxed{x = 200\sqrt{3} \text{ feet}}$$

8. (12 points) Verify the identity

$$\frac{\sec x - \cos x}{\tan x} = \sin x$$

$$\frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \frac{\frac{1 - \cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x}$$

$$= \sin x$$

9. (10 points) Which of the following angles is/are coterminal with θ=300 degrees?

(A) $\pi/3$ $= 60°$ No

(B) $-\pi/3$ $= -60°$ Yes

(C) $4\pi/3$

$\frac{4\pi}{3} \cdot \frac{180°}{\pi} = 240°$ NO

(D) $5\pi/6$ $= 150°$, No

(E) $17\pi/3$

$\frac{17\pi}{3} \cdot \frac{180°}{\pi} = 17 \cdot 60°$

$= 1020°$

$1020° - 360 = 660°$

$660° - 360° = 300°$ Yes

(F) $-5\pi/3$

$\| $

$-300°$, NO

10. (8 points) A circle has radius 5 cm. Calculate the area of the sector cut out by an arc of length 9 cm.(Round your answer to the nearest tenth).

$S = r\theta$

$9 = 5\theta$

$1.8 = \theta$

$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2}(25\text{cm}^2) \cdot 1.8$

$\boxed{A = 22.5\text{ cm}^2}$

Extra Credit(2 points): For an angle θ in Quadrant II, write $\cot\theta$ in terms of $\sin\theta$.

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-\sqrt{1-\sin^2\theta}}{\sin\theta}$

negative since $\sin\theta$ is positive in QII, but $\cot\theta$ is negative.