

Quiz 3A - Math 130

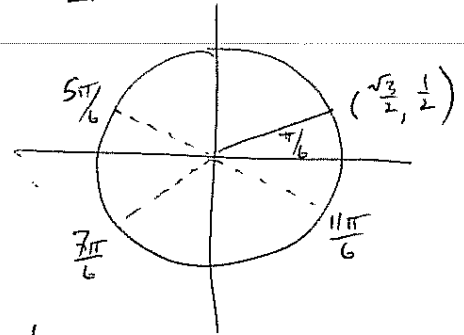
Name: Key

1. (2 points) Find the exact value of all solutions to $\frac{1}{3} - \tan^2 \alpha = 0$

$$\Rightarrow \frac{1}{3} = \tan^2 \alpha$$

$$\pm \sqrt{\frac{1}{3}} = \tan \alpha \quad \checkmark$$

$$\pm \frac{1}{\sqrt{3}} = \tan \alpha$$



$$\alpha = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\alpha = \frac{\pi}{6} + \pi n \quad \text{or} \quad \alpha = \frac{5\pi}{6} + \pi n \quad \text{for } n \in \mathbb{Z}$$

2. (2 points) Find the exact value of all solutions to $\cos 2x - \sin x = 1$ on $[0, 2\pi)$.

$$\Rightarrow 1 - 2\sin^2 x - \sin x = 1$$

~~$$\sin x (2 + \sin x) = 0$$~~

$$-\sin x (2\sin x + 1) = 0$$

$$-\sin x = 0$$

$$\Rightarrow \sin x = 0$$

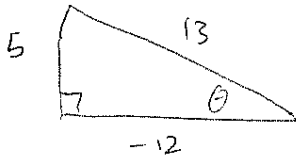
$$x = 0 \quad \text{or} \quad \pi$$

$$\text{or } 2\sin x + 1 = 0$$

$$\Rightarrow \sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6} \quad \text{or} \quad \frac{11\pi}{6}$$

3. (2 points) If $\cot \theta = \frac{-12}{5}$ and $\cos \theta < 0$, find the exact value of $\sin\left(\theta + \frac{\pi}{4}\right)$



$$\sin \theta = \frac{5}{13}$$

$$\cos \theta = \frac{-12}{13}$$

$$\sin\left(\theta + \frac{\pi}{4}\right) = \sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4}$$

$$= \frac{5}{13} \cdot \frac{\sqrt{2}}{2} + \left(\frac{-12}{13}\right) \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{5\sqrt{2} - 12\sqrt{2}}{26}$$

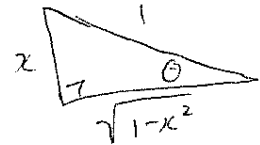
$$= \boxed{\frac{-7\sqrt{2}}{26}}$$

4. (4 points) Give the exact value of:

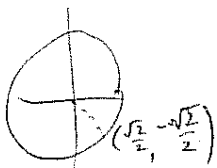
(a) $\cot(\sin^{-1} x)$ Let $\sin^{-1} x = \theta \Rightarrow \frac{x}{1} = \sin \theta$

$$= \cot \theta$$

$$= \boxed{\frac{\sqrt{1-x^2}}{x}}$$



(b) $\tan^{-1}(-1) = \frac{-\pi}{4}$



(c) $\sin^{-1}\left(\sin \frac{7\pi}{6}\right) = \sin^{-1}\left(-\frac{1}{2}\right) = \frac{-\pi}{6}$

(d) $\tan(\sin^{-1}(-1)) = \tan\left(\frac{-\pi}{2}\right)$ is undefined.