

Test 1A, Math of Democracy

Dr. Adam Graham-Squire, Fall 2018

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1. Don't panic.
2. **Show/explain all of your work.** A correct answer with insufficient work will lose points.
3. Read each question carefully, and make sure you answer the the question that is asked. If the question asks for an explanation, make sure you give one.
4. Clearly indicate your answer.
5. Calculators are allowed on this test, but any other technology (cell phones, computers, etc) is NOT allowed.
6. Make sure you sign the pledge.
7. Number of questions = 4. Total Points = 20.

1. (5 points) Consider the following preference schedule:

Number of voters	3	1	1	4	4
1st choice	A	A	A	B	D
2nd choice	B	C	D	A	C
3rd choice	C	D	C	C	B
4th choice	D	B	B	D	A

- (a) Use the Pairwise Comparison method to find the winner of the election (if two candidates are tied in a head-to-head, give each of them a half-point). Briefly show/explain your work.

- (b) Now suppose the the first column of voters decided to NOT show up for the election. The preference schedule now looks like:

Number of voters	1	1	4	4
1st choice	A	A	B	D
2nd choice	C	D	A	C
3rd choice	D	C	C	B
4th choice	B	B	D	A

Run the Pairwise Comparison election on the new preference schedule. What, if anything, does your result tell you about Pairwise Comparison and a Fairness Criterion? Explain your answer.

2. (5 points) Will the following election, under the Instant Runoff method, demonstrate a monotonicity anomaly?

Number of voters	22	8	15	17
1st choice	A	B	B	C
2nd choice	B	A	C	A
3rd choice	C	C	A	B

- If *so*, explain what ballots would need to change, and in what way, to cause the monotonicity anomaly.
- If *not*, explain how changing votes in *any* of the columns could never cause a monotonicity anomaly.

Note: It is fine to reference the criteria for the presence of a monotonicity anomaly in a three-candidate IRV election, but for a complete answer you should actually modify the preference schedule in some way to support your conclusions.

3. (5 points) Below are listed 4 pairs of voting methods and fairness criteria. Choose *two* of the pairs and explain why that particular voting method either *satisfies* or *violates* that particular criteria. Of the two you choose, one of them must be a proof/explanation for how the method satisfies, and the other must be a proof/explanation of how the method violates. If you do more than 2, I will score all of them and give you the highest of the appropriate scores.

- Range voting and Independence of Irrelevant Alternatives criterion
- Sequential Pairwise voting and Condorcet Winner criterion
- Coombs Method and Independence of Irrelevant Alternatives criterion
- Top-two runoff and No-show criterion

4. (5 points) (a) Of all of the voting methods we have looked at so far, which one do you think is the best? Briefly explain why it is better than the other methods.
- (b) Of all of the fairness criteria we have seen so far, which one do you think is *most* important for an election to have? Explain (briefly) why that particular criteria is so important for a fair election.

Extra Credit(1 point) Under any of the voting methods we have seen, is it possible to have a candidate who is not ranked first in *any* ballots, but still wins the election? If so, explain what the method is and give an example of how it could happen. If not, explain why it is NOT possible for at least 3 of the voting methods.

Description of Methods:

- Plurality: Candidate with the highest number of first-place votes wins the election
- Top-two runoff: Eliminate all candidates *except* the two candidates with the most first-place votes. Whoever wins a head-to-head between the top two candidates is the winner of the election.
- Pairwise Comparison: Each candidate does a head-to-head contest against each other candidate. Winning a head-to-head gives a candidate 1 point. Whoever has the most points wins the election.
- Borda Count: Each candidate receives points (in descending order) based on their ranking, for example: 2 points for a first-place vote, 1 point for a second-place vote, and no points for a 3rd-place vote. All the points are totaled and whoever has the most points wins the election.
- Instant Runoff: If no candidate has a majority of first-place votes, the candidate with the fewest first-place votes is eliminated and then there is another round of voting. Rounds continue with candidates being eliminated until one candidate has a majority of the first-place votes.
- Coombs: If no candidate has a majority of first-place votes, the candidate with the most last-place votes is eliminated and then there is another round of voting. Rounds continue with candidates being eliminated until one candidate has a majority of the first-place votes.
- Sequential Pairwise: Candidates are paired up, in a particular order called an *agenda*, to compete in head-to-head contests. If a candidate wins a head-to-head, they advance to the next round. The winner after all of the rounds is the winner of the election.
- Schulze: A graph/network diagram is made showing the candidates, who they beat in head-to-head contests, and by how much. Then each pair of candidates is compared, and the ‘winner’ of each comparison is the candidate with the strongest winning path over the other candidate. The candidate who has a stronger winning path over each of the other candidates is the winner.
- Range: Each voter gives each candidate a score from 0 to 10. Points are tallied and the candidate with the highest score wins.
- Approval: Each voter gives each candidate an approval or not. Approvals are tallied and the candidate with the highest number of approvals wins.

Description of Fairness Criteria:

- Majority Criterion: If a candidate wins a majority of the first-place votes, then that candidate should win the election.
- (upward) Monotonicity Criterion: If you raise a candidate's ranking on some ballots, that should never hurt the candidate (in particular, it should never make the candidate LOSE an election that they otherwise would have won).
- Independence of Irrelevant Alternatives Criterion: Removing a losing candidate from the ballot should never change the winner of an election.
- No-show Criterion: It should never benefit a voter (or group of voters) to NOT cast their ballot.
- Clone-proof Criterion: It should never hurt a candidate to have other *similar* candidates also in the race with them (that is, you should not allow similar candidates to split their vote and end up hurting their chance of winning).
- Condorcet Winner Criterion: If there is a Condorcet *Winner* in a preference schedule (that is, a candidate who beats every other candidate in a head-to-head) then that candidate should win the election.
- Condorcet Loser Criterion: If there is a Condorcet *Loser* in a preference schedule (that is, a candidate who loses to every other candidate in a head-to-head) then that candidate should NOT win the election.