

HNR-1304 – MATHEMATICS OF DEMOCRACY
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VOTING THEORY TEST REVIEW

- For the first test, you should know all the things we have learned so far, including all of the voting methods and fairness criteria. I will include a “cheat sheet” listing all of the methods and fairness criteria on the test, so you should not need to worry about memorizing names of methods or fairness criteria.
- You should also be able to answer questions if given a NEW method or fairness criteria, even if it is one we have not seen before.
- You should be able to justify whether a given voting method satisfies or violates a given fairness criteria. Even if you cannot fully justify or prove an answer, I still expect you to be able to explain your reasoning in a good way (for example, you should NOT use one example to explain why a given method satisfies a fairness criteria when you need a general argument).

On the back side of this sheet are some questions to start to get you thinking about the material we have studied. You should NOT assume that just doing these problems will completely prepare you for the test. You should also NOT assume that any of these questions will be on the test, and should certainly NOT assume that these questions will be exactly the test. You may have had teachers in the past who gave you some test review questions, and then gave you a test that was basically identical to the test review. I am not one of those teachers.

There are also a number of good questions in the textbook that I listed on the “Resources” page, and those are also good for studying for the test.

- (1) Consider the following preference schedule:

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

- (a) Calculate who would be the winner of this election using the following methods. Explain/show your calculation/reasoning for each method.
- Plurality
 - Instant Runoff
 - Pairwise Comparison
 - Borda Count
 - Top-two runoff
 - Schulze method
- (b) If different candidates win using different methods, who do you think should win the election? Explain your reasoning.
- (c) Does this election indicate a violation of any of the fairness criteria (using any of the methods mentioned above). If so, which one(s)? Explain how you know that the election demonstrates that violation.
- (d) Can you find an agenda for sequential pairwise voting (that is where you run successive head-to-head contests in a particular order to determine the winner of the election) in which candidate B would win the election above? How about C , D or A ? If there is no agenda under which a candidate could win, explain why that is the case.
- (2) Of all the methods we have looked at, Instant Runoff is the only one (other than Plurality and Top-two runoff) that is widely used in actual elections, and then only method that actually uses ranking of candidates for voting. Why do you think that is?
- (3) If you were going to propose a ranked-choice voting method for some election system (say, electing the student body president at HPU), which method would you suggest and why? Your answer should be nuanced in the sense of recognizing that your favored method may have flaws, but you should be able to make an argument for why it is superior to other methods.
- (4) Of all of the fairness criteria, which one do you think is the MOST important for an election method to have? Which criterion do you think is the LEAST important?
- (5) Can you give an example of an election (that is, a preference schedule) that would demonstrate how, in a Borda count election, a candidate could be hurt by the introduction of a clone¹? If not, explain why having a clone of oneself could never hurt a candidate in a Borda count election.
- (6) Explain why Range voting satisfies the No-show criterion.
- (7) The Borda count fails the majority criterion. In an example in class we saw that a candidate can win 53% of the first-place vote and still lose the Borda count. The question is, is there a limit to how much a candidate can win the majority of first place votes and still lose in the Borda count? That is, if a candidate wins 60% of the first-place vote, could they still lose the Borda count? If a candidate wins 70% of the first-place vote, could they still lose the Borda count? Explain whether or not you believe there is such a limit, and why. You should probably use some example elections to support your conclusions, but examples by themselves are *not* enough. You will need some general explanations to fill in your overall reasoning.

¹Here a “clone” of a candidate B means a candidate who is similar to B but is maybe not liked quite as much. You can assume that a clone could split the support of a given candidate.