Test 2A, Math of Democracy

Dr. Adam Graham-Squire, Fall 2018

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Don't panic.
2. **Show/explain all of your work.** A correct answer with insufficient work will lose points.
3. Read each question carefully, and make sure you answer the the question that is asked. If the question asks for an explanation, make sure you give one.
4. Clearly indicate your answer.
5. Calculators are allowed on this test, but any other technology (cell phones, computers, etc) is NOT allowed.
6. Make sure you sign the pledge.
7. Number of questions = 5. Total Points = 25.

1. (5 points) Consider the following table of data from a recent election in a state, and answer the questions below.

District	% Democrat	% Republican
1	49	51
2	25	75
3	57	43
4	47	53
5	59	41
6	62	38
7	44	56
8	18	82

$= 45.125$

add

(a) What is the percentage of Democrats, overall, in the state (assume that each district has an equal population)? How many districts did Democrats win in the election? ~~Does that seem to be proportional?~~

(b) What is the uniform partisan shift (from Republicans to Democrats) that would need to occur for Democrats to *win* one more district? If that shift occurred, what would be the new percentage, overall, for Democrats in the state? Explain/show your work.

(c) What is the uniform partisan shift (from Democrats to Republicans) that would need to occur for Democrats to *lose* one more district? If that shift occurred, what would be the new percentage, overall, for Democrats in the state? Explain/show your work.

(a) add up all in Dem column and divide by 8 to get $\frac{361}{8} = 45.125\%$ ✓

Dems win 3 districts (3, 5 and 6), ~~which is $\frac{3}{8} = 0.375\%$ of the seats. Not perfectly proportional, but close.~~

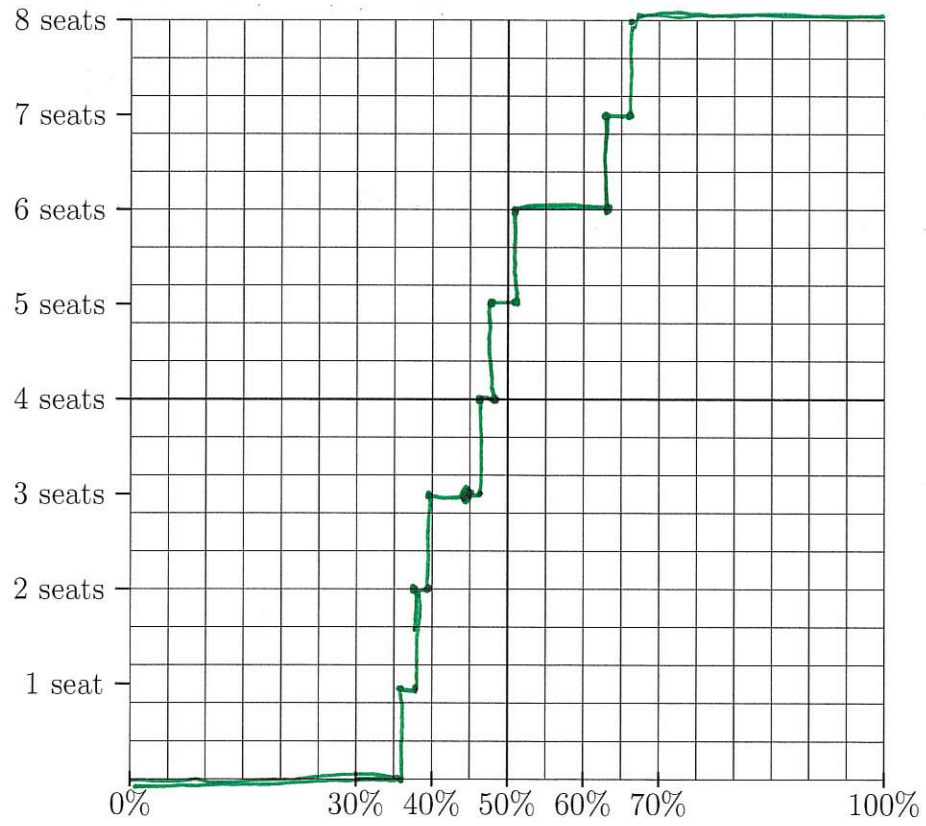
(b) Need to flip district 1, so $\frac{1}{51} = 0.0196 = 1.96\%$. If that occurred, Dem 5 would have $45.125 + 54.875(0.0196) = 46.2\%$ ✓

(c) Need to flip district 3 so $\frac{7}{57} = 0.123 = 12.3\%$. would then have $45.125 - 45.125(0.123) = 39.6\%$ overall

2. (5 points) In question 1 you effectively calculated three points for a partisan symmetry graph for the state. The remaining points are below (you do NOT have to calculate these):

   - When the Democrats have 38.2% of the overall vote, they drop from 2 districts to only win 1 district.
   - When the Democrats have 36.4% of the overall vote, they drop from 1 district to winning no districts.
   - When the Democrats have 48.2% of the overall vote, they increase from winning 4 districts to winning 5 districts.
   - When the Democrats have 51% of the overall vote, they increase from winning 5 districts to winning 6 districts.
   - When the Democrats have 63.4% of the overall vote, they increase from winning 6 districts to winning 7 districts.
   - When the Democrats have 66.5% of the overall vote, they increase from winning 7 districts to winning 8 districts.

   Plot the data above, and the data from question 1, to draw the partisan symmetry graph.



other points are
(39.6, 2)
(45.1, 3)
(46.2, 4)

Answer the question about the partisan symmetry graph on the next page.

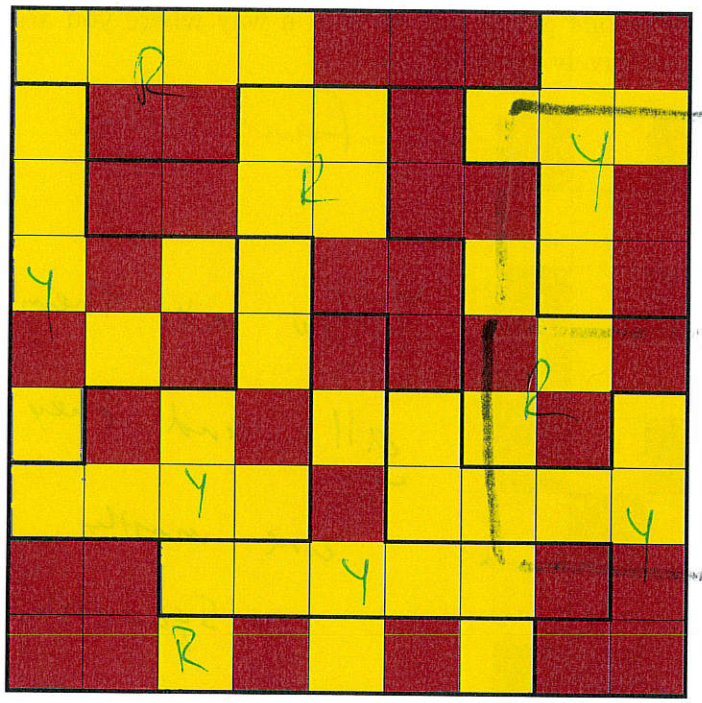
up to -1 for poor explanation

-0.5 if stair steps off and no —

Does your partisan symmetry graph indicate that the state has been gerrymandered? If so, in favor of which party? Use information from the graph to support your conclusions.

The graph looks pretty good, though slightly in favor of the Democrats. It does not pass through the (50, 4) point, but is close. The Democrats are winning 5 seats (→ 62% of seats) when they have less than 50% of the vote, but it isn't until they get above 48%, which is close to 50. There is a long stretch from 51% to 63.4% where Democrats are picking up votes but not seats, which probably indicates packing (in districts 2 and 8)

3. (5 points) Consider the following state of Squaretopia below, with 36 Red squares and 45 yellow squares. Suppose that the state has been districted into the 9 districts on the map.



Red has $\frac{36}{81} = 44\%$ of vote

Yellow has $\frac{45}{81} = 56\%$ of vote

same!

Red wins $\frac{4}{9} = 44\%$ of seats

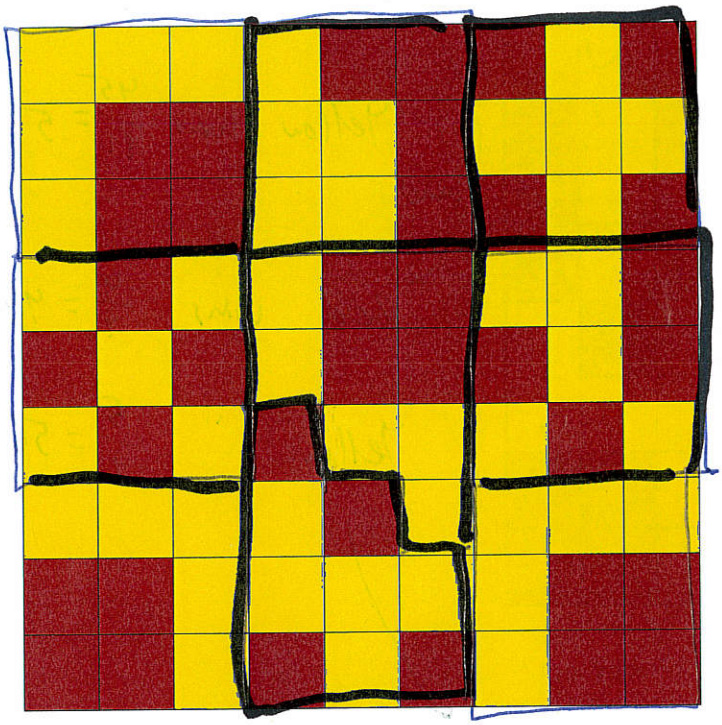
Yellow " $\frac{5}{9} = 56\%$ " "

(2) (a) Do the districts seem fair? Why or why not?

districts are proportional, which is good, but they do seem to be strange (non compact) shapes.

This question is continued on the next page.

(3) (b) Below is the same squaretopia as in question 3. Choose one party (either red or yellow) and try to draw relatively compact districts that favor your chosen party (at least, favor them *more* than the districts given above). In other words, try to gerrymander the state in favor of one party, but do it in a way where you are trying to draw districts that are relatively compact.



Favor Yellow

Yellow wins them all and they are mostly Square ish!

4. (5 points) (a) Calculate the efficiency gap for the district map you drew (assume there are 100 voters in each square).

~~500 to 400 in each square~~ ⇒ Yellow has 56% of vote $V = 0.56$

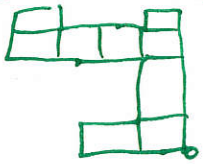
and has all 9 districts ⇒ $S = \frac{9}{9} = 1$

So $2V - S - \frac{1}{2} = 2(0.56) - 1 - 0.5)$

$= 1.12 - 1.5 = \boxed{-0.38}$ which is pretty bad.

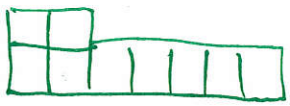
2

(b) ~~(i)~~ Choose the two districts in the original map that you consider to be the *least* compact, and calculate the Isoperimetric score for one of them and the Square Reock score for the other. ~~(ii) Choose the two districts in the *your* map that you consider to be the *least* compact, and calculate the Isoperimetric score for one of them and the Square Reock score for the other.~~



⇒ Iso is $\frac{16A}{P^2} = \frac{16 \cdot 9}{20^2} = \frac{144}{400} = \frac{36}{100} = 0.36$

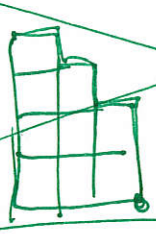
2



⇒ Square Reock is $\frac{9}{7^2} = \frac{9}{49} = 0.18$

(c) Compare your results from questions (a) and (b). What do your answers above indicate about how useful compactness and efficiency gap are at detecting gerrymandering?

(ii)



$I^{50} = \frac{16 \cdot 9}{14^2} = 0.73$

Reock $= \frac{9}{4^2} = 0.56$

1

(c) The E.G. indicates gerrymandering, even though the compactness ~~scores are~~ is better for the maps I drew. This shows that compactness (by itself) may not be a good indicator of gerrymandering (and/or non proportionality)

5. (5 points) At the heart of gerrymandering issue may be an "impossibility" theorem, in the sense that certain desirable traits for a district map may not be compatible with another. That is, it may be impossible to create a perfect district map for a given state. Which of the following aspects (or any others you can come up with) do you think may be incompatible with each other?

choose two that are incompatible, two that are compatible, explain.

- Compactness
- Efficiency Gap
- Proportionality (of votes to seats)
- Partisan symmetry
- Competitiveness of districts
- Ensuring that minority populations have some representation
- Having a map that is not an Outlier.

Your answer does not have to touch on every aspect above, but should be thorough in this test and (if possible) use examples to illustrate your argument.

- There are many answers here. We showed earlier that some maps that may be more proportionally accurate can be much less compact, though those are not necessarily at odds.
- Generally, competitiveness and proportionality will be at odds with each other b/c if you have a lot of competitive districts, they are likely to more heavily favor a party that wins a small majority, giving a large (disproportionate) winners bonus. We saw an example of this in Quiz 4 where there were 5 competitive districts, and we got disproportionate results (one party wins 53% of the vote but 100% of seats). It was symmetric, though so symmetry and competitiveness are not necessarily at odds.
- Generally incompatible: Minority pops and compactness (in fact, many aspects may be incompatible with compactness, such as competitiveness of districts.

Extra Credit(1 point) Outlier methods are not just used in regard to gerrymandering and redistricting. Give another example where an outlier method is used to ascribe validity (or lack thereof) to something.

Many answers. Here is one:

When poll aggregators are trying to figure out which polls (like political polls about which candidate will win a certain race) are good, they often discount the ones that are far outside the average (that is, discount the outliers) as less valid.

Formulas:

1 Compactness measures in Squaretopia

- **Skew measure:** W/L
- **Isoperimetric (Square Polsby-Popper) measure:** $16A/P^2$
- **Square Reock measure:** A/S

2 Real-world Compactness Measures

- **Harris:** W/L
- **Polsby-Popper:** $4\pi A/P^2$
- **Reock:** A/C

3 Efficiency Gap Formulas

- $EG = \dfrac{W_A - W_B}{\text{total votes}}$
- $EG = 2V - S - \dfrac{1}{2}$