Quiz 3A, Math of Democracy

Fall 2019, Dr. Adam Graham-Squire

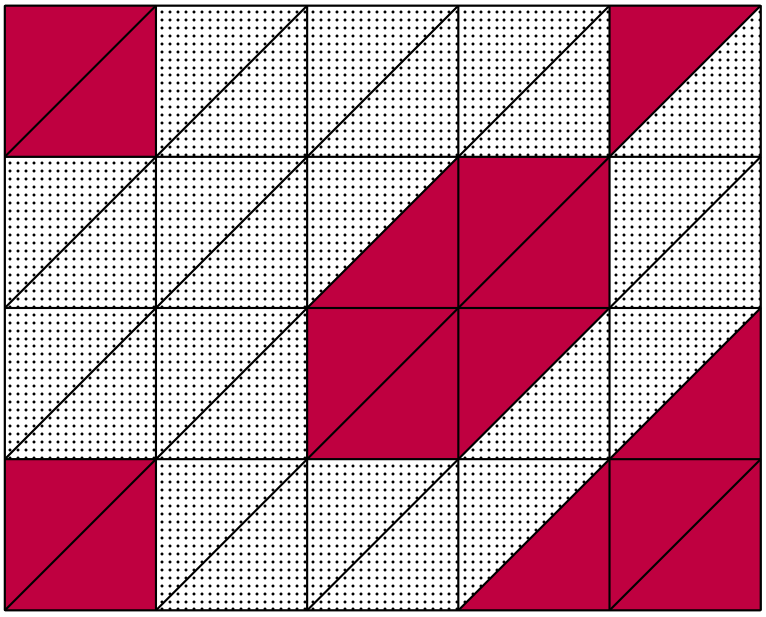
Name: _____

1. (5 points) Consider the state of Trunktopia below[1]. Each precinct is a triangle (40 total precincts), and the state must be divided into 8 equal-sized districts of 5 precincts each. The state is colored based on whether the Purple party or the Dot party has all of the votes in a given precinct.

   (a) Proportionally speaking, what would be a "fair" number of districts for the Purple party to win?

   (b) What is the *maximum* number of districts, theoretically, that the Purple party can win? Explain your reasoning.

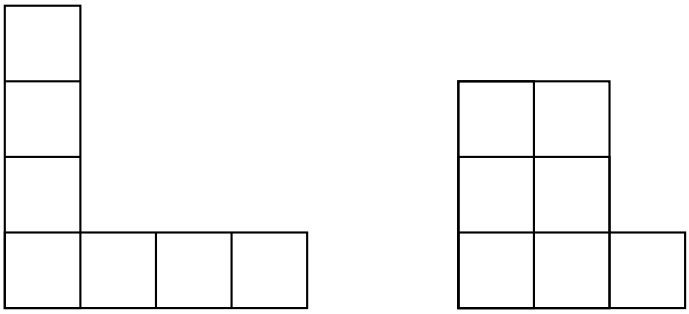
   (c) Practically speaking, can you draw districts so the Purple party actually wins that many districts? Draw the *best* possible districts that you can for the Purple party, and either show that Purple can win their maximum number, *or* explain why Purple cannot achieve their maximum.



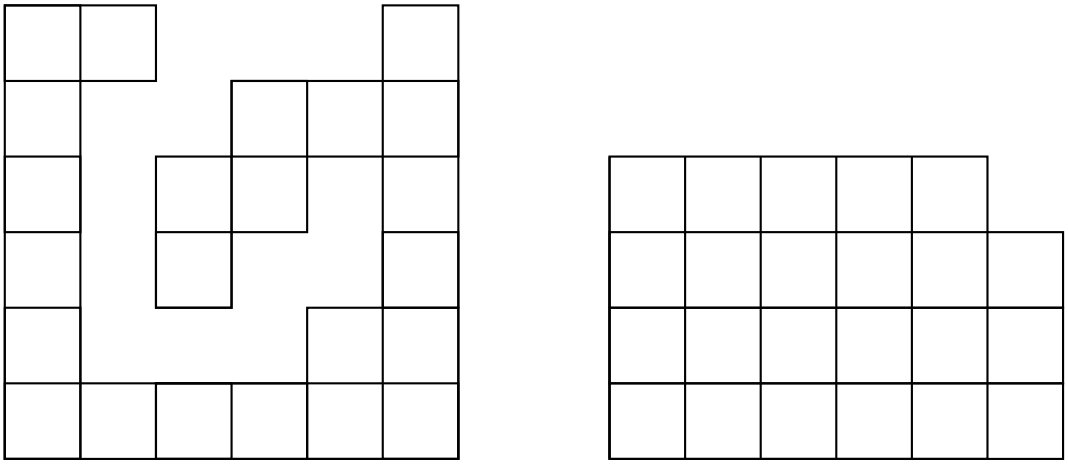
[1] this is where Ronan lived when he was one

2. (5 points) Choose *two* of the three Compactness measures below (Skew, Square Reock, Polsby-Popper) and use the example districts (or keywords) to explain how/why that compactness measure can give less-than-desirable results.

Skew examples:



Square Reock examples:



Real-world Polsby-Popper keywords: Coastlines or Rivers. (feel free to write on the blank back page if you need more space)

Square Compactness measures

- **Skew measure:** W/L, where W is the district's shorter dimension (length or width) and L is its longer one. Equivalently, the width and length of the smallest enclosing rectangle.
- **Isoperimetric (Square Polsby-Popper) measure:** $16A/P^2$, where A is the district's area and P is its perimeter.
- **Square Reock measure:** A/S, where A is the district's area S is the area of the smallest square containing the district.
- **Total perimeter:** Sum of all the perimeters of all districts.

Real-world Compactness Measures

- **Harris:** W/L, where is L is its longest axis and W is its width perpendicular to that axis.
- **Polsby-Popper:** $4\pi A/P^2$, where P is the district's perimeter and A is its area.
- **Reock:** A/C, where A is the district's area C is the area of the smallest circle containing the district.
- **Total perimeter:** Sum of total perimeter of all districts.