

# Quiz 2, Math of Democracy

Fall 2019, Dr. Adam Graham-Squire

Name: Key

The Bucklin method of voting works as follows: All first-place votes are tabulated. If any candidate has a majority of the vote, that candidate wins. If no candidate has a majority, then the second-place votes are added to the first-place votes. If one candidate has a majority, that candidate wins. If multiple candidates have a majority, then whichever candidate has the most votes wins the election. If no candidate has a majority, then third-place votes are included, and the process continues.

Here is an example election:

Number of voters	8	2	4	4	3
1st choice	A	B	C	D	D
2nd choice	C	A	D	B	C
3rd choice	D	D	B	A	B
4th choice	B	C	A	C	A

26 no → j  
 $A: 8 + B = 14$   
 $C: 15$   
 $D: 11$   
 $B = 10$   
 → C wins

In this election, no candidate has a majority of first-place votes (11 or greater). When second-place votes are included, *C* has 15 votes total and *D* has eleven. Both have a majority, but *C* has more, so *C* wins the Bucklin method election.  $(4 + 8 + 3) \rightarrow (4 + 4 + 3)$

It should be clear that the Bucklin method satisfies the Majority criterion.

Now consider the following list of other fairness criteria: Monotonicity (upward or downward), Independence of Irrelevant Alternatives, Clone-proof, No-show, Condorcet (winner or loser). The Bucklin method *violates* at least one of those criteria and *satisfies* at least one. Do the following:

- 1) Choose one criterion that you believe the Bucklin method *fails*, and explain why you think Bucklin fails that criterion.
- 2) Choose one criterion that you believe the Bucklin method *satisfies*, and explain why you think Bucklin satisfies that criterion.

at least 3 out of 5 if doing the correct idea (example for fails, explanation for satisfies)

Max of 4/5 if don't attempt to give an example to show how something fails.

(1) I would guess Bucklin would fail IIA. In part, this is because so many other methods fail IIA, but I also think it for this reason: when an irrelevant candidate drops out, all of the rankings change, so candidates listed 3rd might now be 2nd. That could change the vote totals quite dramatically in the Bucklin method. For example, in the example election on the other side, if B were to drop out as irrelevant, D would get 2 more votes and lose 15-13 to C. While it didn't change the winner, it did get close. (Note: If you change 2nd column to 6 BADL, then C still wins, but removing B makes A the winner  $\Rightarrow$  IIA anomaly)

(2) I think Bucklin satisfies Monotonicity. If a winning candidate is raised in some ballots, that would mean they would have the same # of votes (or more) at each "level" of voting (first-place, 2nd-place, etc). The other candidates would all have the same # of votes or less at each level (because moving the winner up means some losers might move down). Thus the original winner would have the same # or more at each level, against candidates who have the same # or less, so the original winner would still win, at the previous level or earlier.

0 Ex. for IIA

8	6	4	4	3
A	B	C	D	D
C	A	D	B	C
D	D	B	A	B
B	C	A	C	A

total = 26

first place  $\Rightarrow$  no maj.

1 + 2  $\Rightarrow$  A = 14 B = 10 C = 15, D = 11

C wins

remove B  $\Rightarrow$

8	6	4	4	3
A	A	C	D	D
C	D	D	A	C
D	C	A	C	A

A wins

A has 14 first place votes and wins

$\Rightarrow$  IIA anomaly.

~~Satisfies Cond. Winner: Suppose A is Cond. and B wins Bucklin in 2nd round. This means that B~~

Satisfies Cond. Winner: Suppose A is the Cond. winner and B wins over A in Bucklin method. B can't win in round one because then B would be majority winner. Suppose B wins in round 2. Then B would have a majority of votes and more than A, but that can't happen if A is on top of B in a head-to-head!

good, but false reasoning

Fails Crowd. Winner!

A is C.W. but loser in 2nd  
round of Bucklin: A=49

B=53

<u>49</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>45</u>
A	C	D	● D	C
B	B	B	C	D
C	A	A	A	B
D	D	C	B	A