# Quiz 2A, Math of Democracy

Fall 2018, Dr. Adam Graham-Squire

**Name:** Key

Below is a list of pairs of voting methods and fairness criteria. You should choose 3 out of the 4 pairs to answer (you can also answer all 4 and I will grade all four and give you your highest 3 scores). For each pair you answer, you need to decide if the voting method *satisfies* or *violates* the given voting criterion, and then support your conclusion in whatever way is appropriate.

(A) • Borda Count method and the Monotonicity Criterion Sat

(B) • Top-two Runoff method and the Independence of Irrelevant Alternatives Criterion fail

(C) • Instant Runoff method and the Majority Criterion Sat

(D) • Plurality method and the Clone-proof Criterion fail

Max of 1/3.5 if fail/satisfy is incorrect.

(A) Borda Count satisfies the Monotonicity Criterion. If a candidate wins the Borda count, that means they got the most points total. If you move them up in some ballots, that winner will get more points (and some losing candidate(s) will lose points (or stay same)) so the original winner will still have the most points. (-0.5 if no mention of other candidates)

(B) Top-two runoff fails IIA. Consider

| 25 | 22 | 20 | 18 | 15 |
|---|---|---|---|---|
| A | B | C | D | E |
| B | A | D | C | C |
| | | E | E | D |
| | | A | A | A |

A wins Top-two runoff, E is a loser. But if you remove E from all ballots:

| 25 | 22 | 20 | 18 | 15 |
|---|---|---|---|---|
| A | B | C | D | C |
| B | A | D | C | D |
| | | A | A | A |

C is now in top two and beats A in head-to-head, so

(c) IRV satisfies the Majority Criterion b/c ~~all other candidates would drop out~~ if one candidate has a majority, she wins with that majority. Or could say that other candidates will drop with lower # of votes, and majority candidate will win in last round with a majority of the votes.

(d) Plurality fails the Clone-Proof criterion. Same example as for (B):

| 25 | 22 | 20 | 18 | 15 |
|---|---|---|---|---|
| A | B | C | D | E |
| B | A | D | C | C |
| | | E | E | D |

A wins Plurality, but C, D and E are clones of each other and split their vote. If one of C, D was not in the race, those votes would go to a similar candidate (C, D, or E) who would then would win Plurality. Ex: with no E candidate, would have

| 25 | 22 | 35 | 18 |
|---|---|---|---|
| A | B | C | D |
| B | A | D | C |

and C wins the plurality.

-0.5 if no explanation of removing a clone.

# Quiz 2B, Math of Democracy

Fall 2018, Dr. Adam Graham-Squire

≈ 10 min.

**Name:** Key

Below is a list of pairs of voting methods and fairness criteria. You should choose 3 out of the 4 pairs to answer (you can also answer all 4 and I will grade all four and give you your highest 3 scores). For each pair you answer, you need to decide if the voting method *satisfies* or *violates* the given voting criterion, and then support your conclusion in whatever way is appropriate.

(A) • Pairwise Comparison method and the Majority Criterion — satis

(B) • Top-two Runoff method and the Monotonicity Criterion — fail

(C) • Plurality method and the Independence of Irrelevant Alternatives Criterion — fail

(D) • Instant Runoff method and the Clone-proof Criterion — satis

(A) Pairwise Comp. satisfies the Majority Criterion. If you have a candidate with a majority of 1st-place votes, that candidate will have a majority in every head-to-head as well, and thus will be a Condorcet winner and win the pairwise comp. with the most points.

(B) Top-two runoff fails the monotonicity criterion, because you could have a 3rd-place candidate who would beat the 1st-place candidate in a head-to-head. Example:

| 22 | 21 | 20 | 19 | 18 |
|---|---|---|---|---|
| ~~A~~ | B | C | D | E |
| B | A | D | C | C |
| | | E | E | D |
| | | A | A | A |

A wins in top-two against B. But, if 2 BA change to AB you have:

| 24 | 19 | 20 | 19 | 18 |
|---|---|---|---|---|
| A | B | C | D | E |
| B | A | D | C | C |
| | | E | E | D |
| | | A | A | A |

Now A loses to C in the head-to-head b/c B

(C) Plurality and IIA: fails (b/c "irrelevant" candidates can split vote)

Example: Same as for (B)

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| 22 | 21 | 20 | 19 | 18 |
|---|---|---|---|---|
| A | B | C | D | E |
| B | A | D | C | C |

$\Rightarrow$ A wins Plurality, D is a loser

Remove D and get

| 22 | 21 | 20 | 19 | 18 |
|---|---|---|---|---|
| A | B | C | C | E |
| B | A | D | | C |

C now has 39 first-place votes and wins the Plurality election.

(D) IRV and clone-proof: Satisfies, assuming full rankings.

The idea here is that as lower-ranked candidates drop out of the race, they would give their votes to other similar (clone) candidates. So it does not hurt you to have a clone in the same election, b/c if they have fewer votes than you, they drop out before you and give you their votes, so your chance of winning does not get "hurt".