Mat-Rix-Toe: Explaining, Writing and Editing in Linear Algebra

Adam Graham-Squire

High Point University

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Outline





3 Troubleshooting/Things I would change

Intro

- Did not intend to do a writing project.
- $\bullet~\mbox{Game} \rightarrow \mbox{Project} \rightarrow \mbox{Realized in middle I wanted good writing}$
- Has lots of room for improvement.

Description of the game Mat-Rix-Toe

- The board is a square matrix of size 2×2 or larger.
- Two players, the 1-placer and the 0-placer.
- Play is identical to Tic-tac-toe, with 1s instead of Xs, must play until all the entries in the matrix are filled.
- If matrix is invertible, the 1-placer wins. If the matrix is <u>not</u> invertible, the 0-placer wins.

Example Game



Description of Project

- Students learn how to play the game Mat-Rix-Toe, investigate certain properties of the game.
- Specifically, looking for optimal strategy for each player
- Students investigate rule changes and answer some questions.
- Some open-ended questions with no clear answers, students are expected to answer in paragraph format.

Group questions

- What is the optimal strategy in the 2 × 2 case for each player? Assuming each player implements their optimal strategy, who will win? Does it matter who goes first?
- What is the optimal strategy in the 3 × 3 case for each player? Assuming each player implements their optimal strategy, who will win? Does it matter who goes first?
- Suppose the rules were changed so that the 1-placer could put in any number instead of just 1s. Would this change the optimal strategies or general results in the 2 × 2 case or the 3 × 3 case?

Individual questions

- Suppose the rules were changed so that the 1-placer was trying to make the matrix *singular* and the 0-placer was trying to make it *nonsingular*. What would be the optimal strategies in the 2 × 2 case and the 3 × 3 case? Would it matter who goes first?
- Suppose the rules were changed so that the first player gets to put down two entries, then the two players go back and forth each placing only *one* entry each turn after that. How would that change the optimal strategies and overall results for the 2 × 2 and 3 × 3 cases?

Extra Credit questions

- Make preliminary findings for the 4 × 4 case. Make sure that you are using linear algebra concepts to back up your reasoning!
- How does the strategy of playing Tic-Tac-Toe coincide with playing Mat-Rix-Toe? How is the strategy different?

Timeline

Students have 3 weeks, certain parts turned in at one week intervals.

- End of Week 1: Full description of the 2 \times 2 case. Graded for both completion and accuracy.
- End of Week 2: Preliminary findings for the 3 × 3 case. Corrected and returned, grade is based on completion, not accuracy.
- End of Week 3: Complete project due.

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How did they do?

- Students enjoyed learning how to play the game.
- The quality of work varied greatly. Some students put in a lot of time and effort.
- Others not so much: either not putting in sufficient time to come up with correct answers or not correcting the mistakes they had made in earlier explanations.
- Biggest problem: Some students did not have the ability/willingness to explain themselves well in a paragraph format (or otherwise). They did not use mathematics to explain their solutions, instead using vague and imprecise terms to explain themselves.
- Despite my corrections and comments (both on their papers and in front of the class, as well as a list I emailed to all of the students), in their final draft some students still did not explain their results very well.

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Changes for next time around

- Students need more clarity on what is expected in terms of explanation. A rubric, perhaps?
- Give more comments/corrections on students papers, and more opportunities for students to correct and resubmit their work.
- Emphasize that students should probably have matrices, examples, theoretical explanations, and full sentences/paragraphs in their final product. Missing any of those parts usually indicated insufficient work.
- Force students to give at least one play-by-play example of a game being played according to their strategy.

Email I sent my students

- Describe your strategy in as clear and simple terms as possible, as if to someone who has only a rudimentary understanding of linear algebra. Your goal in the explanation is for them to be able to read your explanation and then be able to play the game and win.
- Try to use correct terminology where appropriate (i.e. make a cofactor expansion on the first row and look at the corresponding submatrix).
- Start off by saying who the winner will be in each case, assuming everyone plays their optimal strategy.
- If a certain player will win using their optimal strategy, explain how the other player could win if the first player makes a mistake (usually an example will suffice for this). How many mistakes need to be done for that person to fail?

Email I sent my students (continued)

- Can assume that the first player always plays in the upper left-hand corner.
- When you are doing your explanation, it may help to use a transpose argument, since the determinant of a matrix is the same as the determinant of its transpose.
- The overall goal is to use your strategy explanation to prove your conclusion about who will always win.

Example of Student Work

Example.jpeg



Questions or suggestions for a name other than Mat-Rix-Toe?

Adam Graham-Squire High Point University agrahams@highpoint.edu

This talk and related materials can be found on my website:

http://linus.highpoint.edu/~agrahams/