

Test 3A - MTH 2010
Dr. Graham-Squire, Spring 2015

Name: _____

Key

10:33
11:02

29
→ 25 ✓

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points. Even on multiple choice questions you should show your work—If you get the correct answer but show no work or explanation, you will not receive full points for the problem!
2. Clearly indicate your answer by putting a box around it.
3. Calculators, cell phones and computers are not allowed on this test.
4. Make sure you sign the pledge.
5. Number of questions = 14. Total Points = 70.

1. (5 points)

- (a) (2 points) Harry calculates that $14 \div 3 = 4.2$, which is not correct. What is Harry's mistake, and how would you tell him to correct it?

$14 \div 3 = 4 \text{ rem. } 2$, Harry is putting the remainder as the tenths place, when it should be $4 \frac{2}{3} = 4.\overline{6}$

- (b) Explain why an even number plus an odd number is always odd.

Odd numbers have everything pair up, with one left over. If you add an odd to an even (which has everything already paired) there is nothing to pair with the one left over, so you still have an odd #.

OR $(2n) + (2n+1) = 4n+1 = 2(2n)+1$ is odd

- (c) Which of the following mixtures is more lemony: 2 tablespoons of lemon juice mixed with 3 tablespoons of water, or 5 tablespoons of lemon juice mixed with 7 tablespoons of water? Explain your reasoning.

2:3	2	4	6	8	10	lemon
	3	6	9	12	15	water

5:7	5	10	lemon
	7	14	water

} More lemony b/c it has less water for same amount of lemon

- (d) Is $\frac{3.7}{92}$ a rational or irrational number? Explain.

$\frac{3.7}{92} \cdot 10 = \frac{37}{920}$ which is rational, of form $\frac{p}{q}$.

2. (5 points) Write the expression

$$0.\overline{2} + 0.\overline{41}$$

$$0.\overline{47}$$

as a single fraction, in lowest terms, by

(a) First adding the decimals together, then converting to a fraction (and reducing to lowest terms, if necessary).

(b) First converting each decimal to a fraction, then adding the fractions (and reducing to lowest terms, if necessary).

$$\begin{array}{r}
 \text{(a)} \quad 0.2222 \dots \\
 + 0.4141 \dots \\
 \hline
 0.5363 \dots
 \end{array}
 = 0.\overline{5363} = \frac{63 \div 9}{99 \div 9} = \boxed{\frac{7}{11}}$$

$$\text{(b)} \quad 0.\overline{2} = \frac{2}{9}$$

$$0.\overline{41} = \frac{41}{99}$$

$$\frac{2}{9} + \frac{41}{99} =$$

$$\frac{22}{99} + \frac{41}{99} = \frac{63 \div 9}{99 \div 9} = \boxed{\frac{7}{11}}$$

3. (5 points) Which of the following numbers is divisible by 15 but not by 4?

X (A) 24,342 \rightarrow Not div. by 5

div. by 5 \rightarrow div. by 3

X (B) 68,340 \rightarrow Is div. by 4 ($40/4 = 10$)

X (C) 74,150 \rightarrow Not div. by 3 ($7+4+1+5+0 = 17$)

(D) 24,630 \rightarrow Is div. by 3 and 5, Not div. by 4

b/c 30 is not div. by 4.

4. (5 points) Is 5,083 a prime number? State yes or no, and show/explain how you know that your answer is correct.

5,083 → Not div by 2
→ Not div by 3 (5+8+3+0=16)
→ Not div by 5

$$\begin{array}{r} 726 \\ 7 \overline{) 5,083} \\ \underline{-49} \\ 18 \\ \underline{-14} \\ 43 \\ \underline{-42} \\ 1 \\ \times \end{array}$$

$$\begin{array}{r} 462 \\ 11 \overline{) 5,083} \\ \underline{-44} \\ 68 \\ \underline{-66} \\ 23 \\ \underline{-22} \\ 1 \\ \times \end{array}$$

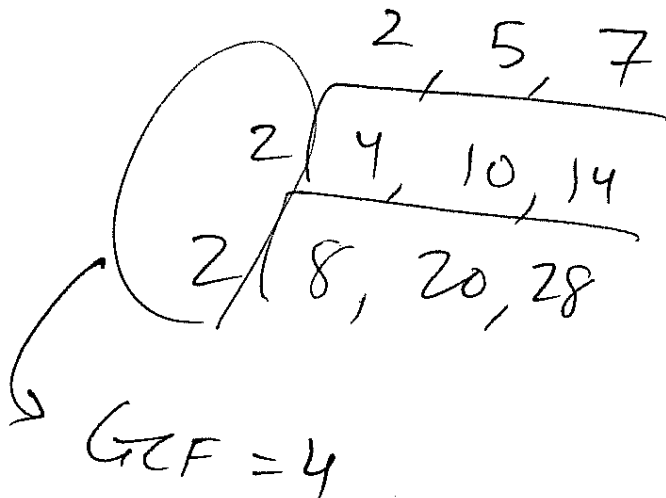
$$\begin{array}{r} 391 \\ 13 \overline{) 5,083} \\ \underline{-39} \\ 118 \\ \underline{117} \\ 13 \\ \underline{-13} \\ 0 \\ \checkmark \end{array}$$

Not prime because 13 is a factor

5. (5 points) A shipping container measures 8 feet by 20 feet by 28 feet. The container is to be filled with identical cube-shaped boxes, each having sides measuring a whole number of feet. What is the largest possible dimensions of the cube-shaped boxes, assuming that you want to fill all of the space in the shipping container? Explain your reasoning!

Need GCF of 8, 20, and 28

~~20~~



⇒ largest dimension is a cube with
each sides 4 feet long

6. (5 points) The prime factorization of a natural number n can be written as $n = p^2 r^2$ where p and r are distinct prime numbers. How many factors does n have, including 1 and itself? Explain/show your work!

(A) 2

(B) 4

(C) 5

(D) 9

(E) 16

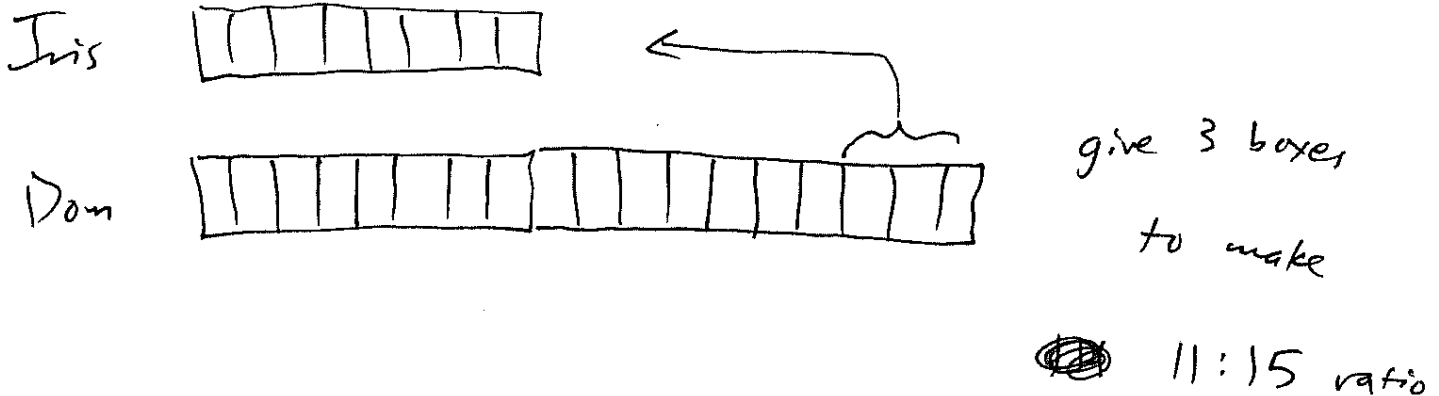
Factor of n :

1	p	p^2
r	pr	$p^2 r$
r^2	pr^2	$p^2 r^2$

9 total

7. (5 points) The ratio of Iris's cats to Dominic's cats is 4:9. Dominic gives Iris 6 cats, and now the ratio of Iris's cats to Dominic's cats is 11:15. How many cats do Dominic and Iris each have now?

4:9 is same ratio as 8:18



So 3 boxes = 6 cats
 \Rightarrow 2 cats per box.

Iris has $11 \times 2 = 22$ cats

Dom has $15 \times 2 = 30$ cats

8. (5 points) Larry is making a sparkling grapefruit punch for a party. To make the punch he adds exactly $\frac{3}{4}$ cup of bubbly water with $\frac{2}{3}$ cup of grapefruit juice. How much grapefruit juice will Larry need if he wants to make 9 cups of punch? Show your work and/or explain your reasoning.

One
batch
of
punch

- (A) Less than 3 cups of grapefruit juice.
(B) More than 3 but less than 4 cups of grapefruit juice.
(C) More than 4 but less than 5 cups of grapefruit juice.
(D) More than 5 cups of grapefruit juice.

$$\frac{3}{4} \text{ cup} + \frac{2}{3} \text{ cup} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} \text{ cups}$$

$$9 \div \frac{17}{12} = 9 \cdot \frac{12}{17} = \frac{108}{17} = 6 \frac{6}{17} \text{ batches in}$$

9 cups

Each batch has $\frac{2}{3}$ cup grapefruit, so do

$$36 \frac{108}{17} \cdot \frac{2}{3} = \frac{72}{17} = 4 \frac{4}{17} \text{ cups of grapefruit juice}$$

9. (5 points) Bob and Jack are filling equal-sized boxes with Easter eggs. Bob can fill up a box in 15 minutes on his own, but it takes Jack 22 minutes to fill a box by himself. Approximate the amount of time it will take for Bob and Jack to fill a box if they work together. You can calculate the exact amount of time if you want, but you do not need to get the exact number in order to get full points. As long as you explain your reasoning (and your reasoning makes sense) and your answer is within 1.5 minutes of the correct time, you will receive full credit.

~~Bob and Jack~~ For 2 Bobs together it would take

$$\frac{15}{2} = 7.5 \text{ minutes.}$$

For 2 Jacks together it would take $\frac{22}{2} = 11$ min.

So it will take between 7.5 and 11 minutes.

Halfway between is about 9.25 minutes so that is my guess.

Precisely: Suppose there are $15 \cdot 22 = 330$ eggs in a box.

Bob does $\frac{330}{15} = 22$ eggs per minute, Jack does $\frac{330}{22} = 15$ eggs per minute. Together they do 37 eggs/min.

Will take $\frac{330}{37} \approx$ 8.9 minutes

$$\begin{array}{r} 8.9 \\ 37 \overline{) 330} \\ \underline{296} \\ 340 \\ \underline{-333} \\ 7 \end{array}$$

10. (5 points) A student uses the following method for dividing fractions: She divides fractions by first finding a common denominator, then dividing the numerators. Here are some examples of her work:

$$\frac{2}{3} \div \frac{3}{4} \rightarrow \frac{8}{12} \div \frac{9}{12} \rightarrow 8 \div 9 = \frac{8}{9}$$

$$\frac{2}{5} \div \frac{7}{20} \rightarrow \frac{8}{20} \div \frac{7}{20} \rightarrow 8 \div 7 = \frac{8}{7}$$

$$\frac{7}{6} \div \frac{3}{4} \rightarrow \frac{14}{12} \div \frac{9}{12} \rightarrow 14 \div 9 = \frac{14}{9}$$

Is her method always, sometimes, or never correct? Explain your reasoning!

Always correct b/c

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$$

and her method is

$$\frac{A}{B} \div \frac{C}{D} = \frac{AD}{BD} \div \frac{CB}{DB} = \frac{AD}{CB}$$

Same

11. (5 points) On a map the distance from Boston to Detroit is 6 cm, and these two cities are 702 miles away from each other. Assuming the scale of the map is the same throughout, which answer below is closest to the distance between Boston and San Francisco on the map, given that they are 2,708 miles away from each other?

(A) 21 cm

(B) 22 cm

(C) 23 cm

(D) 24 cm

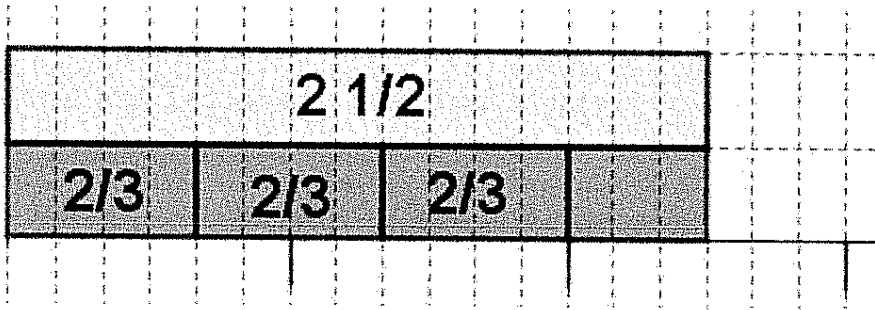
~~6 | 702~~

$$\begin{array}{r}
 7 \\
 10 \\
 \hline
 100 \\
 6 \overline{) 702} \\
 \underline{-600} \\
 102 \\
 \underline{-60} \\
 42 \\
 \underline{-42} \\
 0
 \end{array}$$

$\Rightarrow 1 \text{ cm} = 117 \text{ miles}$

$$\begin{array}{r}
 23.1 \\
 117 \overline{) 2708} \\
 \underline{-234} \\
 368 \\
 \underline{-351} \\
 170 \\
 \underline{-117} \\
 53
 \end{array}$$

12. (5 points) Below is a pictorial representation of $2\frac{1}{2} \div \frac{2}{3} = \frac{5}{2} \div \frac{2}{3} = \frac{5}{2} \cdot \frac{3}{2} = \frac{15}{4} = 3\frac{3}{4}$



Which of the following is the best description of how to find the quotient from the picture? Explain your reasoning to get full points (or at least partial credit if you are not correct).

(A) The quotient is $3\frac{3}{4}$. There are 3 whole blocks each representing $\frac{2}{3}$ and a partial block composed of 3 small rectangles. The 3 small rectangles represent $\frac{3}{4}$ of $\frac{2}{3}$.

(B) The quotient is $3\frac{1}{2}$. There are 3 whole blocks each representing $\frac{2}{3}$ and a partial block composed of 3 small rectangles. The 3 small rectangles represent $\frac{3}{6}$ of a whole, or $\frac{1}{2}$. \Rightarrow ~~X~~ whole is a $\frac{2}{3}$, or 4 small rectangles

(C) The quotient is $\frac{4}{15}$. There are four whole blocks separated into a total of 15 small rectangles. ~~X~~ \Rightarrow should be more than 1

(D) This picture cannot be used to find the quotient because it does not show how to separate $2\frac{1}{2}$ into equal sized groups. ~~X~~

13. (5 points) Wendy wants to do the division $2.112 \div 4.39$, but she can't remember how. She does the calculation

$$2112 \div 439 = 4.8109339\dots$$

correctly, but she is not sure where to place the decimal in the original problem.

- (a) Explain to Wendy what the correct rule is for decimal division, and show her how to use it to get the correct answer.
- (b) Explain to Wendy how she can use her calculation and estimation of the original amounts to figure out where the correct placement of the decimal should go.

(a) correct way is $4.39 \overline{) 2.112}$
 shift decimal two places to the right to get

$$\begin{array}{r}
 439 \overline{) 211.20} \text{ etc... decimal} \\
 \underline{1756} \\
 3560 \\
 \underline{3560} \\
 0
 \end{array}$$

should go
before the 4.

\Rightarrow correct answer is $0.48109339\dots$

(b) $2.112 \approx 2$ and $4.39 \approx 4$ so answer should be close to $2 \div 4 = \frac{2}{4} = 0.5$,
 so put decimal in front of 4

$\Rightarrow 0.48109339\dots$

14. (5 points) Use the problem below to answer the question that follows:

T-shirts are on sale for 20% off. Tasha paid \$8.73 for a shirt. What is the regular price of the shirt? There is no tax on clothing purchases under \$175.

Let p represent the regular price of these t-shirts. Which of the following equations is correct?

(A) $0.8p = \$8.73$

(B) $\$8.73 + 0.2 \times \$8.73 = p$

(C) $1.2 \times \$8.73 = p$

(D) $p - 0.2 \times \$8.73 = p$

$0.8p = 80\%$ of original price
 $= \text{Original price} - 20\%$

so $0.8p = 8.73$

Extra Credit(2 points): Arrange the following terms in a tree (or some other format, if you would prefer) to indicate which sets include the others: rational numbers, counting numbers, irrational numbers, prime numbers, whole numbers, real numbers and integers. For example, if I gave you the list: animals, mammals, bee, dog, you would put animals at the top, then mammals and bee below it, then dog below mammals (because bees, dogs and mammals are all included in "animals", and dogs are included in "mammals", but bees are not.

of real #s
 19

