MTH-2010, SPRING 2015
DR. GRAHAM-SQUIRE

TEST 2 REVIEW ANSWERS

Below are brief answers to each problem, most of which would need a much longer explanation in order to receive full credit, the short answer here is just to help you know that you are on the right track. Some questions have also been answered with videos, if the written answer seems unsatisfactory.

- Section 3.2:
  - #9, **Ans**: The student is just adding numbers to 88, first to get to the nearest ten, then hundred, in order to get up to 201. The student then adds up all the "added" numbers to see how much was added total, which is the answer to 201-88. The problem with their equations, though, is that they are not showing equal things. The first equation starts with 88+2 and ends with 201, but 88+2 $\neq$ 201, so they do not show true equations. A more mathematically accurate way of showing the solution could be

    $$\begin{aligned} 88 &+ 2 = 90 \\ 90 &+ 10 = 100 \\ 100 &+ 100 = 200 \\ 200 &+ 1 = 201 \end{aligned}$$

    Then end with $2 + 10 + 100 + 1 = 113$.
  - #11, **Ans**: (a) see video. (b) Tylisha's method is valid because subtraction is giving you the *distance* between two numbers. So if you add the same amount to each each number, you keep the distance between them the same, but you have simplified your calculation a little bit.

    (c) Yes, her method will work with other problems. See video for an example.
- Section 3.3:
  - #5, **Ans**: Zachary may not have realized that you should carry ten tenths to equal one unit. So he added 4 plus 7 and then wrote the 11 to the right of the decimal point, instead of writing 1 and carrying 1. I would explain to Zachary that ten tenths is equal to one whole, so eleven tenths is equal to one whole (unit) and one-tenth. Then I would show him how we demonstrate this by carrying the one to get the correct answer.
  - #11, **Ans**: (a) Erin is wrong. Her mistake is that when she borrows one from the hour (the 1), she adds 100 minutes to the next column instead of adding 60 minutes. See the video for a full explanation.

    (b) Another way to solve the problem is to add up to the next hour. So add 15 minutes to 9:45 to get to 10 o'clock. Then add an hour to get to 11 o'clock, then another 30 minutes to get to 11:30. Total we added up 15+30= 45 minutes and 1 full hour, for a total of 1 hour and 45 minutes, which is the correct answer.

- Section 3.4:
  - #11, **Ans**: Arnold is wrong because he is confusing the wholes. His first diagram is bad because he is representing a whole with both one square (for the first two squares) and then with 3 squares (to show the 2/3). In his answer, he puts them together and represents the whole with 5 squares.
  - #14, **Ans**: Problem 1: No, need to know if any beads are both pink and oblong (if there is no overlap, then it would be a correct calculation. If there are beads that are both pink and oblong, it would not be the correct calculation). Since there is not enough information, the question cannot be answered.

    Problem 2: No, because the 1/5 is of the remaining beads (not the total number of beads), so it would be (3/4)*(1/5) are non-pink and oblong. Thus the total number that are either pink or oblong would be 1/4 + (3/4)*(1/5)=1/4 + 3/20=5/20 + 3/20=8/20=2/5.

    A problem that *can* be solved by 1/4 +1/5: "One-fourth of Sarah's beads are pink. None of the pink beads are oblong. One-fifth of Sarah's beads are oblong. What fraction of Sarah's beads are either pink *or* oblong?"
  - #21, **Ans**: Square 1: 1/16, because if you cut the rectangle in half lengthwise and height wise, you would have four smaller squares, each one-fourth of the big square. The upper right hand corner would be half of a small square, so (1/4)*(1/2)=1/8. The green triangle is exactly half of that corner, so (1/8)*(1/2)=1/16.

    Square 2: The answer is 37/48. Here, I would calculate the white areas and subtract. The upper white triangle is one-half of the top one-third of the square, so it has area of (1/2)(1/3=1/6. The bottom right white triangle is one-fourth of the rightmost fourth of the square, so it has area of (1/4(1/4)=1/16. Thus the total white area is 1/6 + 1/16= 8/48 +3/48 = 11/48. Subtracting that from the whole gives 1- 11/48 = 48/48 - 11/48= 37/48, which is the answer.
  - #23, **Ans**: a) No, the wholes may be different, because you don't know how many miles of road there are in each county. Even if they had the same number of miles, though, you would NOT add the percentages, because your whole changes from a one-county area to a two-county area.

    (b) 30% of 200 is 60 miles, and 10% of 800 miles is 80, for a total of 140 miles out of a total of 1000 miles, which is 14%. For the vice versa question, you get 10% of 200 is 20 miles, and 30% of 800 miles is 240, for a total of 260 miles out of a total of 1000 miles, which is 26%.

    (c) You need to know how many miles are in each county. If the number of miles is the same, then Ming is right, otherwise Ming is wrong.
- Section 3.5: #3, **Ans**: Should get -3, 3, -3, and 1, using a number line to do it. See video for full answer.

- Section 4.1, #8, #9 (Full answers to these are on the Section 4.1 video). For #8, Trey and Miles are both correct, but John is wrong. For #9, the answers are (a) 1560, (b) 780, and (c) they are different because the order of president and vice president matter, so they are each counted separately, but the order of co-presidents does not mater, so you have to divide by two in order to account for the "doubles".

- Section 4.2: #3, **Ans**: 10,000 is equal to 10*10*10*10, so you move one place for each ten. 100,000 is just 5 tens multiplied together.
- Section 4.3:
  - #10, **Ans**: break the 16 into 4*4 and 25 into 5*5, then move the numbers with the associative property so that you can do 4*5 twice. This would look like:

$$16 \times 25 = (4 \times 4) \times (5 \times 5) = 4 \times (4 \times 5) \times 5 = 4 \times (20 \times 5) = 4 \times 100 = 400$$

  - #13, **Ans**: $n \times 4 = n \times (2 \times 2) = (n \times 2) \times 2$. So multiplying by 4 is the same as first multiplying by two, then multiplying by two again.
  - #16, **Ans**: 36*240=8,640 total square inches. Since there are 12 ladybugs on 6 square inches of the design (3 times 2=6), that means that there are 2 ladybugs per square inch. This gives a total of 8640*2=17280 ladybugs. Another way to do it is that since the swatch is 2 inches tall, you can fit 18 swatches from top to bottom (36 inches divided by 2) on the wrapping paper. Since the swatch is 3 inches long, you can fit 80 swatches from left to right (240 inches divided by 3) on the wrapping paper. This means you can put a total of 18*80=1440 swatches on the wrapping paper. Each swatch has 12 ladybugs on it, so 1440*12=17280 total ladybugs.
- Section 4.4:
  - #5, **Ans**: $6 \times (2+3) = 6 \times 2 + 6 \times 3$. You can either count the total number of people sitting in front, then the total number of people sitting in back $(6 \times 2 + 6 \times 3)$ or count the total number of people in each car, then multiply by the number of cars $(6 \times (2+3))$. See video for a more thorough explanation.
  - #9, **Ans**: (a) draw a 12 by 15 array and block out a 10 by 10 square and a 2 by 5 rectangle, show Ted that this does NOT fill up the entire array. See video for more thorough explanation.
    (b) $12 \times 15 = (10+2) \times (10+5) = 10 \times 10 + 10 \times 5 + 2 \times 10 + 2 \times 5$, which is 175, more than 110. Ted's mistake was that his method did not account for the inner terms of the FOIL operation when you do the distributive property.
  - #14, **Ans**: $(10,000,000,000 - 1)(10,000,000,000 - 1) = 10^{20} - 10^{10} - 10^{10} - 1 = 99,999,999,979,999,999,999$. Explanation: I wrote each 999,999,999 as $10^{10} - 1$, then FOILed out the expression. $10^{20} - 1$ is 19 9's, and then I subtracted 2 from the 10 billion's place to account for subtracting the two 10^{10}'s.
  - #16, **Ans**: (b) is larger because you multiply 1, 2, etc. by a larger number (1000001 instead of 1000000) each term, so your total at the end will subsequently be larger as well.
- Section 4.5:
  - #7, **Ans**: You will get the same discount either way, due to the distributive property. If the costs of your items were $A, B, C, D, andE$, she did $0.9(A+B+C+D+E)$, the other way would be $0.9A + 0.9B + 0.9C + 0.9D + 0.9E)$, both of which are equal by the distributive property.

  - #12, **Ans**: $30\%(240) = 30\%(100+100+40) = 30+30+(30\%(10+10+10+10)) = 30+30+3+3+3+3 = 72$
  - #15, **Ans**: (a) She is basically realizing that 2/5 is equal to 1/2 minus 1/10. Since 1/10 is equal to 1/5 of 1/2, she find 1/2 (which is 630), then finds 1/10 (which is 126), and does the subtraction to get the answer. A string of equations demonstrating how the distributive property is used would be:

$$\frac{2}{5} \times 1260 = \frac{4}{10} \times 1260 = (\frac{1}{2} - \frac{1}{10})1260 = \frac{1}{2} \times 1260 - \frac{1}{10} \times 1260 = 630 - 126 = 504$$

    (b) Terrell is basically seeing that 2/5 is equivalent to 4/10, so to find 2/5 of 1260 he is multiplying 1260 by four (by doubling twice) and then dividing by 10.

$$\frac{2}{5} \times 1260 = \frac{4}{10} \times 1260 = \frac{2 \times 2 \times 1260}{10} = \frac{2 \times 2520}{10} = \frac{5040}{10} = 504$$

- Section 4.6: #8, **Ans**: Break the array into 4 parts, a 20 by 20 square, a 3 by 20 rectangle, a 20 by 7 rectangle, and a 3 by 7 rectangle. Each of those areas correspond to a part of the partial-products algorithm. In the standard algorithm the 7 by 23 rectangle corresponds to one part and the 20 by 23 rectangle to the other part. See video for a fuller explanation (with diagram).

- Section 5.1:
  - #6, **Ans**: (a) The problem cannot be solved because we don't know how many initial brownies there were, and did any of the frosted brownies get hearts? And in any case, who puts frosting on brownies? And how would the hearts stick to the brownies if there was no frosting on them? Would you use tape?

    (b) It is NOT a problem for 1/3 times 1/4, because it cannot be solved. A modification of the problem that would fit (1/3)*(1/4) is: "You put white frosting on 1/3 of the brownies, and then put small red hearts on 1/4 of the frosted brownies. What fraction of the brownies have both frosting AND hearts?"

    (c) No, it is not, because we don't know if there is overlap between the frosted and hearted brownies. A modification of the problem that would fit (1/3)+(1/4) is: "You put white frosting on 1/3 of the brownies, and then put small red hearts on 1/4 of the brownies that are NOT frosted. What fraction of the brownies have either frosting OR hearts?"
  - #15, **Ans**: The square is split into two halves, one on the left and the other on the right. The half on the left is divided into sixths, 5 of which are shaded, so this gives us (5/6)(1/2). On the right half, the shaded triangle takes up half of the space, which you could see by dividing it down the middle from top to bottom, then realigning the shaded pieces to show half of the space is shaded. This represents (1/2)(1/2). When we add them together we get (5/6)(1/2)+(1/2)(1/2)=5/12 +1/4= 5/12+3/12=8/12=2/3
- Section 5.2:
  - #3, **Ans**: 2.43 is between 2 and 3, and 0.148 is between 0 and 1, so the answer must be between 0 and 3. The only way to do that is to put the decimal in front of the three to get 0.35964. Another way of looking at it is to say that 2.43 is between 2 and 3,

so if you double or triple 0.148 you will get a number bigger than 0.148, but still less than 1, which would imply 0.35964 as the answer.

- #9, **Ans**: (a) It is 25.4-(0.25 $\times$ 25.4), so neither is right.
  (b) To do 0.25*25.4, you could have the original scenario but ask how much shampoo did Katie use? To do the subtraction, you could have Katie use 0.25 fluid ounces, instead of 0.25 of the bottle, then ask how much is left.

- Section 5.3: #2, **Ans**: (a) can occur: 0.2*0.1=0.02, (b) can occur: 0.1*20=2, (c) can occur: -4*2=-8, (d) can occur: 0.1*(-20)=-2, (e) cannot occur: a negative times a positive is always negative, which would be less than the positive number.
- Section 5.4:
  - #5, **Ans**: No, 2 to any power would be even, so it could not end with a 5. The E17 means $\times 10^{17}$, and you can't tell what the exact number would be because it is in scientific notation with the decimal number truncated due to limitations on the number of digits on the calculator.
  - #12, **Ans**: a six-digit number is always $N \times 10^5$ and an eight-digit number would be $M \times 10^7$. So you would get $N \times M \times 10^{12}$. If M times N is less than ten, then you would have 10^{12}, which would lead to a 13-digit number. But it M times N is greater than or equal to 10, you would add another place and have a 14 digit number.

- Section 6.1:
  - #5, **Ans**: (a) Yes. You have zero cows, which you want to divide equally amongst 5 friends. How many cows does each friend get? Answer: zero.
    (b) No. You have 5 cows, and you want to put zero cows in each bucket. How many buckets will you need? This has no answer, because no matter how many buckets you have, if you are putting nothing in them then you will never use up your 5 cows.
  - #8, **Ans**: (a) (i) $10^5 \div 10^2 = \frac{10^5}{10^2} = \frac{100000}{100} = 1000 = 10^3$, Similar calculations give (ii) 10^2, and (iii) 10^1.
    (b) When you subtract the second power from the first power, you get the power for the answer.
    (c) $10^A \div 10^B$ is the same thing as A copies of ten on top of a fraction and B copies of 10 on the bottom. You can then cancel 10's from top and bottom, and you will end up canceling B 10's from the top (because A is bigger), leaving you with $A - B$ 10's on the top and just a 1 on the bottom, which is equal to 10^{A-B}.
- Section 6.2:
  - #5, **Ans**: (a) 19 1/3 miles makes sense.
    (b) need 4 road crews, fraction of a road crew does not make sense.
    (c) 3 remainder 6, give each kid 3 and have 6 left over, does not make sense to break pencils into fractions.
    (d) 1 3/4 or 1.75, would split up the packs to eat as much as possible.
  - #13, **Ans**: Work backwards as follows: there are 100 left, which represents two out of three parts. So divide by two to get 50, then multiply by 3 to get 150–this is how many coins were there before the 3rd robber divided them and took a third for himself. Now add 2 to the 150 to account for the hush money and that means there was 152

coins left after the second robber got done dividing and taking. Repeat this process two more times to get an answer of 347. Notationally, the calculation looks like:

$$347 = \frac{\left(\frac{(\frac{100}{2} \times 3)+2}{2}\right) \times 3 + 2}{2} \times 3 + 2$$