Mat-Rix-Toe Project

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Linear Algebra, Spring 2012

For this project, you will learn how to play the game Mat-Rix-Toe and then investigate certain properties of the game. In particular, you will be looking for an optimal strategy for each player to win. You can work by yourself or in pairs, although there are certain questions that must be answered individually even if you work with someone else. You will have almost 3 weeks to complete the assignment, though certain parts must be turned in at one week intervals. Specifically, the following due dates will hold:

- Friday, Feb. 24: You must turn in a paper stating who is in your group (if you are working in a group) and a full description of the 2×2 case (that is, question 1). This part will be graded for both completion and accuracy.
- Friday, Mar. 2: Your group must turn in your preliminary findings for the 3×3 case (question 2). I will correct it, give comments and return it to the group. Your score for this part will be based on completion, not accuracy.
- Friday, Mar. 16: Complete project (Questions 1 through 3 for the group, individual questions and any extra credit) due at the beginning of class.

Description of the game Mat-Rix-Toe:

The board is a square matrix of size 2×2 or larger. There are two players, the 1-placer and the 0-placer. First a coin is flipped to decide who goes first. Suppose the 1-placer goes first. The 1-placer starts by putting a 1 as an entry somewhere in the matrix. Then the 0-placer puts a zero somewhere, then the 1-placer goes, etc, back and forth until all the entries in the matrix are filled. If the matrix is nonsingular, the 1-placer wins. If the matrix is singular, the 0-placer wins.

Group questions:

1. What is the optimal strategy in the 2×2 case for each player? Assuming each player implements their optimal strategy, who will win? Does it matter who goes first? Make sure to justify your answers.
2. What is the optimal strategy in the 3×3 case for each player? Assuming each player implements their optimal strategy, who will win? Does it matter who goes first? Make sure to justify your answers.
3. Suppose the rules were changed so that the 1-placer could put in *any* number instead of just 1s. Would this change the optimal strategies or general results in the 2×2 case or the 3×3 case?

Individual questions. If you are working in a pair, you have to split up and one person will do one question, the second person will do the other. If you are working on the project individually, you can choose which question you want to work on and ignore the other. Make sure you clearly state your results and justify your conclusions:

4. Suppose the rules were changed so that the 1-placer was trying to make the matrix *singular* and the 0-placer was trying to make it *nonsingular*. What would be the optimal strategies in the 2×2 case and the 3×3 case? Would it matter who goes first?

5. Suppose the rules were changed so that the first player gets to put down two entries, then the two players go back and forth each placing only *one* entry each turn after that. How would that change the optimal strategies and overall results for the 2×2 and 3×3 cases?

   Extra Credit questions(these are also individual): Answer one or both of these questions to get up to five extra credit points.

   (a) Make preliminary findings for the 4×4 case. Make sure that you are using linear algebra concepts to back up your reasoning!

   (b) How does the strategy of playing Tic-Tac-Toe coincide with playing Mat-Rix-Toe? How is the strategy different?

Score Breakdown:

- 2×2 case, due Feb. 24: 10% (Graded on completion and accuracy)

- Initial results for 3×3 case, due Mar. 2: 10% (Graded solely on completion)

- Completed Project, due Mar. 16: Group portion (that is, the assigned problems)- 60%; Individual portion- 20%. (Graded for both completion and accuracy)

Additional Comments:

- You can come to my office hours if you need help or have questions. If you are working with a group, though, you should all come together as a group (and if that means you need to schedule a time to meet with me that is not at my normal office hours time, that is fine).

- If you are working in a group, I expect each member of the group to write up at least one of the solutions that are turned in at the end. The individual portions, obviously, should be in your own handwriting.

- Late projects will lose 10 percent each day they are late. Projects turned in over three days late will not be accepted.

- It may help to just play the game for a while and keep track of who wins to get an idea of what an optimal strategy would be.