MTH 2310, SPRING 2012

MINITEST 1 REVIEW
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Answers

- The test will take 30 minutes.
- You can use a calculator, but you will not need one. You must still show your work, though, even if you can do a problem on the calculator (e.g. there will be questions asking you to row reduce).
- The test will cover sections 1.1-1.5.
- To study for the test, I recommend the following:
  (1) Looking over your notes and trying to rework old problems from class, HW problems, and quizzes.
  (2) Before the exercises at the end of each section, there are also Practice Problems that are solved after the exercises. Try to do these.
  (3) You can also work out problems from the Supplementary Exercises at the end of Chapter 1, in particular numbers 1(a-p), 5, 7, 9, 11, 13. The answers to most of those are in the back of the textbook if you want to check your work.
  (4) Look at the materials from last semester that are posted to blackboard. Not all problems are ones that are covered by our minitest, but many are (Quizzes 1 and 2, and some of Test 1).
- As with the quizzes, it is important that you know not just the answer to a question, but also how to explain your answer.

Some problems to work on in class today (these are taken from the supplementary exercises at the end of Chapter 1):

(1) True/False: If True, justify your answer with a brief explanation. If False, give a counterexample or a brief explanation.

F b. Any system of n linear equations in n variables has at most n solutions.

T f. If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.

F j. The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.

T m. If an $n \times n$ matrix has n pivot positions, then the reduced echelon form of A is the $n \times n$ identity matrix (the "identity matrix" is the square matrix with diagonal entires of 1 and zeroes everywhere else).

T o. If A is an $m \times n$ matrix, if the equation $A\mathbf{x} = \mathbf{b}$ has at least two different solutions, and if the equation $A\mathbf{x} = \mathbf{c}$ is consistent, then the equation $A\mathbf{x} = \mathbf{c}$ has many solutions.

(2) Solve the system of linear equations. If the system has an infinite number of solutions, write the solutions in parametric vector form. Show your work!

$$4x_1 + 8x_2 + 12x_3 = 36$$
$$2x_1 - x_2 + x_3 = 8$$
$$3x_1 - x_3 = 3$$

Ans: $x = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

(3) (Number 2 in Supp. Ex.) Let a and b be real numbers. Describe the possible solution sets of the linear equation $ax = b$. Hint: the number of solutions depends on the values of a and b.

- $a \neq 0$, b is anything $\Rightarrow$ Unique solution $(x = \frac{b}{a})$
- $a = 0$, $b \neq 0$ $\Rightarrow$ No solution
- $a = 0$, $b = 0$ $\Rightarrow$ Infinite # of solutions.

(4) (Number 8 in Supp. Ex): Describe the possible echelon forms of the matrix A if
(a) A is a 2×3 matrix whose columns span $\mathbb{R}^2$ $\longrightarrow$ $\begin{bmatrix} * & * & * \\ 0 & * & * \end{bmatrix}$, $\begin{bmatrix} * & * & * \\ 0 & 0 & * \end{bmatrix}$, or $\begin{bmatrix} 0 & * & * \\ 0 & 0 & * \end{bmatrix}$
(b) A is a 3×3 matrix whose columns span $\mathbb{R}^3$.

$\hookrightarrow \begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$ only one possibility.

(5) Construct a 3×3 matrix A with all nonzero entries, and a vector $\mathbf{b}$ in $\mathbb{R}^3$ such that $\mathbf{b}$ is *not* in the set spanned by the columns of A.

Many answers. Need to have a matrix A that reduces to have a row of zeros.

Ex. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\qquad b = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$

(6) Let A be a matrix and $\mathbf{w}$ a vector such that $A\mathbf{w} = \mathbf{0}$. Show that for any scalar c, the vector $c\mathbf{w}$ is also a solution to the equation $A\mathbf{x} = \mathbf{0}$ (that is, show that $A(c\mathbf{w}) = \mathbf{0}$).

$$A(cw) = c(Aw) = c \cdot 0 = 0$$