Quiz 7, Linear Algebra

Fall 2017, Dr. Adam Graham-Squire

Name: ______________________________________

1. (3 points) Let V and W be vector spaces and $T : V \to W$ a linear transformation. Let H be a nonzero subspace of V, then $T(H)$ is a subspace of W given by the images of vectors in H. Prove that dim $T(H) \leq$dim H. (Hint: If $\mathbf{g}$ is in $T(H)$, that means that $\mathbf{g} = T(\mathbf{h})$ for some $\mathbf{h}$ in H. Define a basis for H, then show that the images of that basis will span $T(H)$. Then explain why that proves what you want it to prove.)

2. (3 points) Determine the dimensions of Nul A and Col A for $A = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$. Briefly explain your reasoning.

3. (4 points) Use an inverse matrix to find $[\mathbf{x}]_{\mathcal{B}}$ for $\mathbf{x} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \end{bmatrix} \right\}$.