Quiz 5, Linear Algebra

Dr. Adam Graham-Squire

4.5 min ⇒ 20 min in da[y]

Name: Key

1. (4 points) Calculate $\det A$ for $A = \begin{bmatrix} 4 & -7 & 3 & 0 & -5 \\ 0 & 3 & 0 & 0 & 0 \\ 7 & -6 & 4 & 4 & -8 \\ 5 & 5 & 2 & 0 & 0 \\ 0 & 9 & -1 & 0 & 2 \end{bmatrix}$.

$\det A = -0 + 3\det\begin{bmatrix} 4 & 3 & 0 & -5 \\ 7 & 4 & 4 & -8 \\ 5 & 2 & 0 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} - 0 + 0 - 0$

$= 3\left[0 - 4\det\begin{bmatrix} 4 & 3 & -5 \\ 5 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix} + 0 - 0\right]$

$= 3\left(-4\left(4\cdot 2\cdot 2 + 0 + (-5)(5)(-1) - 0 - 0 - (3\cdot 5\cdot 2)\right)\right)$

$\begin{matrix} 4 & 3 & -5 & 4 & 3 \\ 5 & 2 & 0 & 5 & 2 \\ 0 & -1 & 2 & 0 & -1 \end{matrix}$

$= -12(16 + 25 - 30)$

$= -12(11) = \boxed{-132}$

2. (3 points) Let A and P be square matrices, with P invertible. Show that $\det(PAP^{-1}) = \det A$.

$$\det(PAP^{-1}) = \det(P) \cdot \det(A) \cdot \det(P^{-1})$$

$$= \det P \cdot \det(P^{-1}) \cdot \det A$$

$$= \cancel{\det P} \cdot \frac{1}{\cancel{\det P}} \cdot \det A$$

$$= \det A$$

3. (3 points) Can a square matrix with two identical columns be invertible? Why or why not? Make sure you explain your reasoning.

<u>No</u>. Two identical columns $\Rightarrow$ columns of the matrix are linearly dependen

$\Rightarrow$ Matrix is <u>not</u> invertible (by I.M.T.)