Quiz 4, Linear Algebra

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5 min

$\Rightarrow$ 15 $\to$ 20?

Name: Key

1. (3 points) Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make $AB = BA$?

$AB = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} = \begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix}$

$BA = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 23 & 15 \\ 6-3k & 15+k \end{bmatrix}$

$\Rightarrow$ Need $-9 = 6-3k$ and $-10+5k = 15$

$-15 = -3k$ $\qquad$ $5k = 25$

$\boxed{5 = k}$ $\qquad$ $\boxed{k = 5}$

2. (2 points) Set up and explain how to find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$. You do not have to actually find the inverse (if it exists), just set up the calculation and explain what you would do to find the inverse (and what you would get if it was not invertible).

Augment $[A \,\vdots\, I]$ with the 3×3 Identity matrix and row reduce. If you get to RREF and have

$\left[\begin{array}{ccc:c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \text{stuff} \\ 0 & 0 & 1 & \end{array}\right]$ Then this is A^{-1} and A is invertible. If the first 3×3 is not the I_3 matrix, then A is not invertible.

3. (2 points) Let $\mathbf{u} = \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute (a) $\mathbf{v}^T\mathbf{u}$ and (b) $\mathbf{v}\mathbf{u}^T$.

$$v^T u = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix} = \boxed{-3a + 2b - 5c}$$

$$v u^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} -3 & 2 & -5 \end{bmatrix} = \boxed{\begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}}$$

4. (3 points) Explain why the columns of an $n \times n$ matrix A are linearly independent when A is invertible. (Note: you cannot just say "because of the Invertible Matrix Theorem", you must give an explanation).

If the columns are l.in. indep that means there are no free variables $\Rightarrow$ pivot in every column $\Rightarrow$ n pivots.

If you have n pivots, you will be able to row reduce to the identity matrix, so you will be able to get $[A \,\vdots\, I] \xrightarrow{\text{reduce to}} [I \,\vdots\, A^{-1}]$

So A will be invertible.