Quiz 3, Linear Algebra

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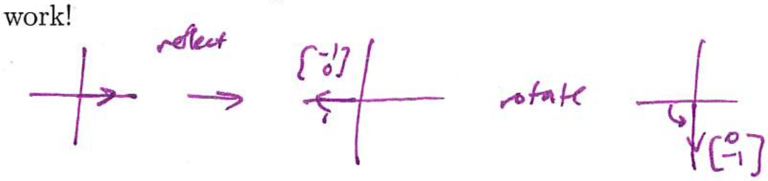
5 min

$\Rightarrow$ 15 $\to$ 20 min.

Name: Key

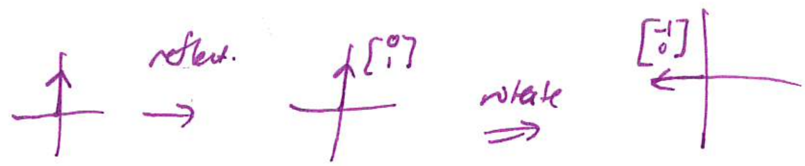
1. (4 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points through the vertical x_2-axis (or y-axis) and then rotates points $\frac{\pi}{2}$ radians counterclockwise. Find the standard matrix of T. Show your work!

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$



$= \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) =$



$= \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$\Rightarrow T = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

2. (3 points) Given $A = \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 5 & 3 & 2 \\ -4 & 1 & -5 \\ -4 & -1 & -3 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$, observe that the first column is the sum of the second and third columns. Without performing any row operations to reduce the matrix, find a nontrivial solution to $A\mathbf{x} = \mathbf{0}$. [Hint: Write $A\mathbf{x} = \mathbf{0}$ as a vector equation.]

Note that $$1a_1 - 1a_2 - 1a_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \vec{0}$$

$$\Rightarrow A \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = 0$$

$\vec{x} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ is a nontrivial solution.

3. (3 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent. (Recall: a set $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is linearly dependent if there exist $c_1, c_2, \ldots, c_n$, not all zero, such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n = \mathbf{0}$).

$\{v_1, v_2, v_3\}$ lin. dep $\Rightarrow$ There exist c_1, c_2, c_3 not all zeo

such that $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$ "Mult" by T

$\Rightarrow T(c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3) = T(\vec{0})$ Prop. of lin. transf.

$\Rightarrow c_1 T(\vec{v}_1) + c_2 T(\vec{v}_2) + c_3 T(\vec{v}_3) = \vec{0}$

$\Rightarrow$ there exist c_1, c_2, c_3 such that

$\Rightarrow \{T(\vec{v}_1), T(\vec{v}_2), T(\vec{v}_3)\}$ ~~are~~ is lin. dep. by definition.