Quiz 9, Linear Algebra

Fall 2017, Dr. Adam Graham-Squire

6:15 → 25 min.

Name: Key

1. (4 points) Let $\mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}$. Describe the set H of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ that are orthogonal to $\mathbf{v}$. (Hint: you will need to look at two cases, one where $\mathbf{v} = 0$ and one where $\mathbf{v} \neq 0$.)

Need all $\begin{bmatrix} x \\ y \end{bmatrix}$ such that $\begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\Rightarrow ax + by = 0$ ✓

Case 1: if $v = \vec{0}$ then $a = 0$ and $b = 0 \Rightarrow$ x and y can be anything. So in this case, ✓ $H =$ all of $\mathbb{R}^2$.

Case 2: if $v \neq \vec{0}$, then either $a \neq 0$ or $b \neq 0$. If ✓ $a \neq 0$, then ~~$b = \frac{-ax}{y}$~~ $x = \frac{-by}{a}$ and y is free ✓

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = y \begin{bmatrix} -b/a \\ 1 \end{bmatrix}$, so $H =$ all scalar multiples of $\begin{bmatrix} -b/a \\ 1 \end{bmatrix}$

if $b \neq 0$, then $y = \frac{-ax}{b} \Rightarrow H =$ all scalar multiples of $\begin{bmatrix} 1 \\ -a/b \end{bmatrix}$, which is the same thing.

2. (4 points) Let $\mathbf{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$. Write $\mathbf{y}$ as the sum of two orthogonal vectors, one in Span$\{\mathbf{u}\}$ (usually denoted as $\hat{\mathbf{y}}$) and one orthogonal to $\{\mathbf{u}\}$.

$$\hat{y} = \frac{y \cdot u}{u \cdot u}(\vec{u}) = \frac{-9}{61}\begin{bmatrix} 5 \\ -6 \end{bmatrix} = \boxed{\begin{bmatrix} -45/61 \\ 54/61 \end{bmatrix}}$$ 0.5

✓ ✓

$$z = y - \hat{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} - \begin{bmatrix} -45/61 \\ 54/61 \end{bmatrix} =$$

0.5 ✓

$$= \begin{bmatrix} 3\,45/61 \\ 3\,7/61 \end{bmatrix} = \begin{bmatrix} \frac{228}{61} \\ 190/61 \end{bmatrix}$$

3. (2 points) Determine if the vectors are orthonormal. If they are orthogonal but *not* orthonormal, normalize them to be orthonormal.

$$\begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix}$$

$$u \cdot v = \frac{-2}{9} + \frac{2}{9} = 0 \implies \text{orthogonal}$$ ✓

$$\|u\| = \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{1} = 1$$ ✓

to normalize

$$\|v\| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \frac{\sqrt{5}}{3} \neq 1, \text{ so do } \frac{1}{\left(\frac{\sqrt{5}}{3}\right)}\begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix}} \text{ for}$$ ✓

↳ so not orthonormal!