Test 3, Linear

Name: ________________________________

ID Number: ________________________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work. A correct answer with insufficient work will lose points.
2. Read each question carefully and make sure you answer the question that is asked. If the question asks for an explanation, make sure you give one.
3. Clearly indicate your answer by putting a box around it.
4. Calculators are allowed on this exam.
5. Make sure you sign the pledge and write your ID on both pages.
6. All questions are required.
7. Number of questions = 8. Total Points = 100.

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1. (16 points) True or False: If true, briefly explain why. If false, explain why or give a counterexample.

   (i) A change of coordinates matrix is always invertible.

   (ii) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{0}$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

   (iii) One way to find the eigenvalues of a square matrix A is to use row operations to reduce A to upper triangular form, then the diagonal entries will be your eigenvalues.

   (iv) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^3$. If the distance from $\mathbf{u}$ to $\mathbf{v}$ equals the distance from $\mathbf{u}$ to -$\mathbf{v}$, then $\mathbf{u}$ is orthogonal to $\mathbf{v}$.

2. (20 points) Multiple Choice Questions. You do not need to show any work to receive full credit for these problems (although showing work can get you partial credit if your answer is wrong).

   (a) The trivial solution in $\mathbb{R}^3$ is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. The subspace spanned by it has dimension

   (a) 0 (b) 1 (c) 2 (d) 3

   (b) The vector space of polynomials of degree 4 or less is $\mathbb{P}_4$, it has dimension

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 5

   (c) If the null space of a 3×4 matrix is 2-dimensional, what is the dimension of the column space of A?

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

   (d) If A is a 7×4 matrix, what is the largest possible rank of A?

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 5 (g) 6 (h) 7

   (e) If A is a 6×5 matrix, what is the smallest possible dimension of the nullspace of A?

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 5 (g) 6

3. (15 points) Diagonalize the matrix $\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$.

4. (10 points) Suppose A is a matrix such that $A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Show that the only eigenvalue of A is 0. (Hint: Suppose that $A\mathbf{v} = \lambda\mathbf{v}$ for some vector $\mathbf{v}$. Now calculate $A^3\mathbf{v}$ in two different ways to conclude that $\lambda = 0$.)

5. (14 points) Let $A = \begin{bmatrix} 1 & -3 & 1 & -1 \\ 2 & -6 & 3 & 1 \\ -1 & 3 & -5 & -5 \end{bmatrix}$. An echelon form for A is $\begin{bmatrix} 1 & -3 & 1 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ and its reduced echelon form is $\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Answer the following (Show work if necessary):

(a) Find the dimension of and a basis for Nul A.

(b) Find the dimension of and a basis for Col A.

(c) Find the dimension of and a basis for Row A.

(d) Find the rank of A.

6. (12 points) Consider the following set of four polynomials in $\mathbb{P}_2$:

$$\{2 + t + 2t^2, 3 + 2t - 2t^2, 2 + 2t, 1 + 2t - t^2\}$$

Answer the following:

(a) Is the set linearly independent?

(b) Does the set span $\mathbb{P}_2$?

(c) Is the set a basis for $\mathbb{P}_2$? If not, how could you modify it to make it a basis?

7. (10 points) Let W= Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Show that if $\mathbf{x}$ is orthogonal to each of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$, then $\mathbf{x}$ is orthogonal to every vector in W.

8. (3 points) Let $\mathbf{v} = \begin{bmatrix} -4 \\ 3 \\ -3 \\ \sqrt{2} \end{bmatrix}$. Find a unit vector in the direction of $\mathbf{v}$.

Extra Credit (2 points): Find by inspection an eigenvalue and its corresponding eigenvector for the matrix $\begin{bmatrix} 1 & 0 & 2 & 4 & -3 \\ 2 & 0 & 4 & -3 & 1 \\ -1 & 0 & 1 & 0 & 4 \\ 1 & 0 & 1 & 1 & 1 \\ 2 & 0 & 2 & 3 & -3 \end{bmatrix}$.