MTH 2310, FALL 2011

TEST 3 REVIEW

- The test will take the full period.
- You can use a calculator, but you will not need one.
- The test will cover sections 4.3-4.6, 5.1-5.3, and 6.1.
- To study for the test, I recommend looking over your notes and trying to rework old problems from class, HW problems, and questions from previous quizzes. You can also work out problems from the Supplementary Exercises at the end of Chapters 4, 5 and 6. In particular:
  - (i) Chapter 4 Supplementary: # 1 (d, e, f, h-s), 2, 5, 6-11.
  - (ii) Chapter 5 Supplementary: # 1 (a-r), 2, 3, and 6(a).
  - (iii) Chapter 6 Supplementary: # 1 (a-h) and the assigned HW problems for that section.

  The answers to most of those are in the back of the textbook if you want to check your work. I will post answers to the even numbered ones as well on blackboard.
- As with the quizzes, it is important that you know not just the answer to a question, but also how to explain your answer.

Some problems to work on in class today (most of these are even-numbered problems from the textbook):

(1) True/False: If True, justify your answer with a brief explanation. If False, give a counterexample or a brief explanation.
  - (a) A linearly independent set in a subspace H is a basis for H.
  - (b) Let $\mathcal{B}$ be a basis for V and $P_{\mathcal{B}}$ the change-of-coordinates matrix. Then $[\mathbf{x}]_{\mathcal{B}} = P_{\mathcal{B}}\mathbf{x}$ for all $\mathbf{x}$ in V.
  - (c) The number of free variables in the equation $A\mathbf{x} = \mathbf{0}$ equals the dimension of Nul A.
  - (d) If A and B are row equivalent, then their row spaces are the same.
  - (e) The eigenvalues of a matrix are on its main diagonal.
  - (f) A row replacement operation on a matrix A does not change the eigenvalues.
  - (g) If A is invertible, then A is diagonalizable.
  - (h) $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$.

(2) Consider the polynomials $\mathbf{p}_1(t) = 1 + t$, $\mathbf{p}_2(t) = 1 - t$, and $\mathbf{p}_3(t) = 2$. By inspection, write a linear dependence relation among $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$. Now find a basis for Span$\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$.

(3) Are the polynomials $(1 - t)^3$, $(2 - 3t)2$, and $3t^2 - 4t^3$ linearly independent in $\mathcal{P}_3$? Do they form a basis?

(4) Consider the following subspace of $\mathbb{R}^4$: $\left\{ \begin{bmatrix} 3a+6b-c \\ 6a-2b-2c \\ -9a+5b+3c \\ -3a+b+c \end{bmatrix} \right\}$. Find a basis for the subspace and state its dimension.

(5) Let H be an n-dimensional subspace of an n-dimensional vector space V. Explain why $H = V$.

(6) Is it possible that all solutions of a homogeneous system of ten linear equations in twelve variables are multiples of one fixed nonzero solution? Explain your answer in terms of rank as well as the dimension of things such as the column, null, and/or row spaces.

(7) Find a basis for the eigenspace of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}$ corresponding to the eigenvalue $\lambda = -2$.

(8) Explain why A and A^T have the same characteristic polynomial (assume A is a square matrix).

(9) Diagonalize $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$.

(10) Let $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$. (a) Calculate $\dfrac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{w} \cdot \mathbf{w}}\mathbf{w}$. (b) Find a unit vector orthogonal to $\mathbf{x}$ (Hint: first find an orthogonal vector by inspection and then normalize it).

(11) Suppose $\mathbf{y}$ is orthogonal to $\mathbf{u}$ and $\mathbf{v}$. Show that $\mathbf{y}$ is orthogonal to every vector in Span$\{\mathbf{u}.\mathbf{v}\}$. (Hint: An arbitrary $\mathbf{w}$ in Span$\{\mathbf{u}.\mathbf{v}\}$ is of the form $\mathbf{w} = c_1\mathbf{u} + c_2\mathbf{v}$.)