Test 1, Linear

Name: ________________________________

ID Number: ________________________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work. A correct answer with insufficient work will lose points.
2. Read each question carefully and make sure you answer the the question that is asked. If the question asks for an explanation, make sure you give one.
3. Clearly indicate your answer by putting a box around it.
4. Calculators are allowed on this exam.
5. Make sure you sign the pledge and write your ID on both pages.
6. Number of questions = 8. Total Points = 100.

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1. (10 points) For each question, state if it is True or False. If True, briefly justify why it is true. If False, give an explanation or a counterexample to show why it is false.

   (a) Any augmented matrix with a bottom row of all zeroes represents a system that has an infinite number of solutions.

   (b) If a consistent system of linear equations has no free variables, then it has a unique solution.

   (c) If v_1 is in span$\{v_2, v_3\}$, then v_2 is in span$\{v_3, v_1\}$.

   (d) If B is an $m \times n$ matrix with m pivots, then the linear transformation $T(\mathbf{x}) = B\mathbf{x}$ is a one-to-one mapping.

2. (15 points) For the system of equations

$$\begin{aligned} 2x_1 + 4x_2 - 2x_3 &= 6 \\ -x_1 - 3x_3 &= 3 \\ 2x_2 - 4x_3 &= 0 \end{aligned}$$

do the following:

(a) Write the augmented matrix.

(b) Row reduce the matrix to reduced row echelon form.

(c) Find all solutions and write your answer in parametric vector form (if solutions exist).

3. (10 points) Let B be a 5×3 matrix, let $\mathbf{y}$ be a vector in $\mathbb{R}^3$, and let $\mathbf{z}$ be a vector in $\mathbb{R}^5$. Suppose that $B\mathbf{y} = \mathbf{z}$. Is $B\mathbf{x} = 4\mathbf{z}$ consistent? Explain your answer.

4. (15 points) Determine all values of p and q such that the system of equations

$$\begin{aligned} 2x_1 + px_2 &= 2 \\ 3x_1 + x_2 &= q \end{aligned}$$

has

(a) No solution.

(b) Infinitely many solutions.

(c) A unique solution.

5. (10 points) Are the vectors $v_1 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$ linearly dependent or linearly independent? Explain your answer.

Does $\{v_1, v_2, v_3\}$ span $\mathbb{R}^2$? What about $\mathbb{R}^3$ or $\mathbb{R}^4$? Explain.

6. (15 points) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & 1 \end{bmatrix}$, $u = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ -11 \\ 2 \end{bmatrix}$, $c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and define a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(x) = Ax$.

(a) Find $T(u)$.

(b) Find all x in $\mathbb{R}^2$ whose image under T is b.

(c) Is c in the range of T? Explain your answer.

7. (15 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the following transformation: T first lengthens vectors by a factor of 3, then projects vectors to the x_2 axis (a projection to the x_2 axis does the following- it sends x_1 to zero, and x_2 to x_2). Answer the questions below about T:

(a) For an arbitrary $\mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, what is $T(\mathbf{x})$?

(b) Show that T is a linear transformation.

(c) Find the standard matrix of T.

(d) Is T an onto map? Is T one-to-one? Explain.

8. (10 points) If $A = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ 3 & 0 \end{bmatrix}$, find the matrix AB.

Extra Credit(2 points) Suppose A, B and C are such that AB and CA are defined. If AB is a 2×3 matrix and CA is a 3×4 matrix, what are the respective sizes of A, B and C?