MTH 2310, FALL 2011

TEST 1 REVIEW

- The test will take the full period.
- You can use a calculator, but you will not need one.
- The test will cover sections 1.1-1.5, 1.7-1.9, and 2.1
- To study for the test, I recommend looking over your notes and trying to rework old problems from class, HW problems, and questions from previous quizzes. You can also work out problems from the Supplementary Exercises at the end of Chapter 1, in particular numbers 5, 7, 9, 13, 15, 17, 20, 21, and 25 and all of the true/false. The answers to most of those are in the back of the textbook if you want to check your work.
- As with the quizzes, it is important that you know not just the answer to a question, but also how to explain your answer.

Some problems to work on in class today (these are taken from the supplementary exercises at the end of Chapter 1):

(1) True/False: If True, justify your answer with a brief explanation. If False, give a counterexample or a brief explanation.

b. Any system of n linear equations in n variables has at most n solutions.

f. If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.

j. The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.

m. If an $n \times n$ matrix has n pivot positions, then the reduced echelon form of A is the $n \times n$ identity matrix.

o. If A is an $m \times n$ matrix, if the equation $A\mathbf{x} = \mathbf{b}$ has at least two different solutions, and if the equation $A\mathbf{x} = \mathbf{c}$ is consistent, then the equation $A\mathbf{x} = \mathbf{c}$ has many solutions.

r. If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are not in $\mathbb{R}^2$

v. If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, then $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

y. If A is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbb{R}^5$ onto $\mathbb{R}^6$.

(2) Let a and b be real numbers. Describe the possible solution sets of the linear equation $ax = b$. Hint: the number of solutions depends on the values of a and b.

(3) (Number 8 in text): Describe the possible echelon forms of the matrix A if
  (a) A is a 2×3 matrix whose columns span $\mathbb{R}^2$
  (b) A is a 3×3 matrix whose columns span $\mathbb{R}^3$.

(4) (Number 14 in text): Determine the value(s) of a such that $\left\{ \begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} a \\ a+2 \end{bmatrix} \right\}$ is linearly independent.

(5) (Number 22 in text): Let A be a 3×3 matrix with the property that the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^3$. Explain why the transformation must be one-to-one.