Quiz 7, Linear

11:00
$\approx$ 11:05
5 min.

Name: Key

1. (3 points) The set $M_{2\times 2}$ is the vector space of all two-by-two matrices with the standard operations of addition and scalar multiplication. Determine if the subset H of all matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is a subspace of $M_{2\times 2}$.

• Zero vector: let $a=b=d=0$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is in H. ✓

• Addition: $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} + \begin{bmatrix} s & t \\ 0 & v \end{bmatrix} = \begin{bmatrix} a+s & b+t \\ 0 & d+v \end{bmatrix}$ is in H ✓

• Scalar Mult.:

$$c\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} ca & cb \\ 0 & cd \end{bmatrix} \text{ is in } H \checkmark$$

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2 min

2. (4 points) Although a matrix A is not necessarily square, the matrices A^TA and AA^T are always square. Justify that in general $\det(A^TA) \neq (\det AA^T)$ by doing the following: Let $A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Choose easy values for a, b, and c and then calculate $\det(A^TA)$ and $\det(AA^T)$.

$A = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

$$A^TA = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1^2 + 0^2 + 2^2 \end{bmatrix} = \begin{bmatrix} 5 \end{bmatrix}$$

$$\det(A^TA) = 5$$

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$$AA^T = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

$\det(AA^T) = 0$ because has a row of zeros.

So $\det(A^TA) \neq \det(AA^T)$

3. (3 points) Find an explicit description of Nul A by listing vectors that span the null space if $A = \begin{bmatrix} 1 & -3 & 0 & 2 & 0 \\ 0 & 0 & 1 & -7 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & -7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow x_1 - 3x_2 + 2x_4 = 0$$
$$x_3 - 7x_4 = 0$$
$$x_5 = 0$$
$$x_2 = x_2$$
$$x_4 = x_4$$

$$x_1 = 3x_2 - 2x_4$$
$$x_2 = x_2 + 0$$
$$x_3 = 0 + 7x_4$$
$$x_4 = 0 + x_4$$
$$x_5 = 0 + 0$$

$$x = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ 7 \\ 1 \\ 0 \end{bmatrix}$$

So $\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 0 \\ 7 \\ 1 \\ 0 \end{bmatrix}$ span the null space.