Quiz 5, Linear

11:35
11:39
4 mins
⇒ give 15 in class

Name: Key

1. (4 points) Suppose $CA = I_n$. Show that the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Explain why A cannot have more columns than rows.

$$Ax = 0$$
$$\Rightarrow CAx = C \cdot 0$$
$$\Rightarrow I_n \cdot x = 0$$
$$\Rightarrow x = 0 \quad \text{which is the trivial solution.}$$

- If $Ax=0$ has only the trivial solution, there are no free variables so A cannot have more columns than rows.

2. (2 points) Let $\mathbf{u} = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute $\mathbf{u}\mathbf{v}^T$ and $\mathbf{u}^T\mathbf{v}$.

$$uv^T = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} -a & -b & -c \\ 3a & 3b & 3c \\ -2a & -2b & -2c \end{bmatrix}$$

$$u^T v = \begin{bmatrix} -1 & 3 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1a + 3b - 2c \end{bmatrix}$$

3. (4 points) Let $A = \begin{bmatrix} 1 & 4 \\ 3 & 13 \end{bmatrix}$. Find A^{-1} and use it to solve the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{13-12} \begin{bmatrix} 13 & -4 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -4 \\ -3 & 1 \end{bmatrix} \quad \checkmark$$

$$Ax = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\Leftrightarrow \quad A^{-1}(Ax) = A^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \checkmark$$

$$\Leftrightarrow \quad x = \begin{bmatrix} 13 & -4 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$