

Linear - Quiz 1

12:15
12:22
7

Name: Key

1. (4 points) Determine if the system is consistent. You do not need to completely solve the system, but you should explain your reasoning (briefly) for why the system is consistent or not.

$$\begin{array}{rclcrcl} x_1 & & +3x_3 & & = & 2 \\ & x_2 & & -3x_4 & = & 3 \\ & -2x_2 & +3x_3 & +2x_4 & = & 1 \\ 3x_1 & & & +7x_4 & = & -5 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array} \right] \checkmark$$

$-3R_1 + R_4 \rightarrow R_4$
 $2R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{array} \right] \checkmark$$

$3R_3 + R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & -5 & 10 \end{array} \right] \checkmark$$

Consistent. This \uparrow is in REF and has 4 pivot

positions, (no ~~0000~~ 0000) so it is consistent.



2. (6 points) Choose x and y such that the system has

- (a) No solution
- (b) A unique solution
- (c) Infinitely many solutions

$$\begin{bmatrix} 1 & x & : & 3 \\ 2 & 10 & : & y \end{bmatrix}$$

correction

Explain your reasoning.

$$-2R_1 + R_2 \Rightarrow R_2 = \begin{bmatrix} 1 & x & : & 3 \\ 0 & 10-2x & : & y-6 \end{bmatrix} \quad \checkmark$$

(a) No solution if bottom row is $00 : \#$, where $\# \neq 0$

$$\begin{array}{l} \text{So need } 10-2x=0 \Rightarrow x=5 \quad \checkmark \\ \text{and } y-6 \neq 0 \Rightarrow y \neq 6 \quad \checkmark \end{array}$$

(b) Unique solution if bottom row has pivot in the 2nd column

$$\Rightarrow 10-2x \neq 0 \Rightarrow x \neq 5 \quad \checkmark$$

y can be anything.

(c) Infinitely many if bottom row is all zeroes

$$\Rightarrow 10-2x=0 \quad \text{and} \quad y-6=0$$

$$\Rightarrow \begin{array}{l} x=5 \quad \text{and} \quad y=6 \\ \checkmark \quad \quad \quad \checkmark \end{array}$$