Test 2 - MTH 1420

Dr. Graham-Squire, Spring 2012

1:23

Name: Key

ID Number: ____________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on all parts of the test, however for questions labeled NO CALCULATOR you must take the integral by hand and show all of your work. On those questions your should only use your calculator to confirm that your answer is correct.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you want to do trig substitution, the rules are:
   - If we have a factor of the form $\sqrt{a^2 - x^2}$, we do the substitution $x = a\sin\theta$.
   - If we have a factor of the form $\sqrt{a^2 + x^2}$, we do the substitution $x = a\tan\theta$.
   - If we have a factor of the form $\sqrt{x^2 - a^2}$, we do the substitution $x = a\sec\theta$.
6. Make sure you sign the pledge and write your ID on both pages.
7. Number of questions = 7. Total Points = 75.

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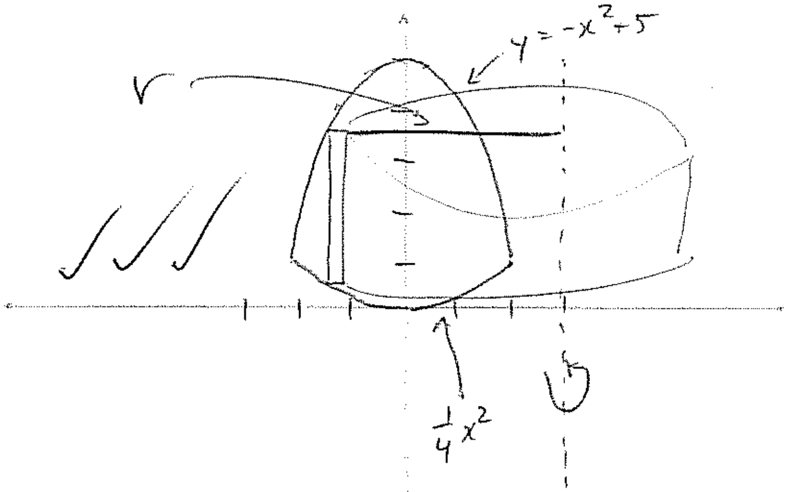
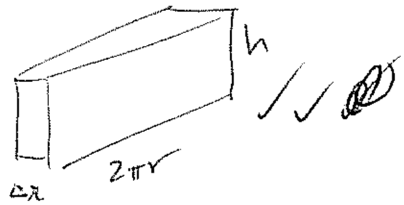
Test 2

1. (12 points) Let R be the region enclosed by the curves $y = \frac{1}{4}x^2$ and $y = -x^2 + 5$. Set up but do not integrate an integral that represents the volume generated when R is rotated about the line $x = 3$.





$r = 3 - x$

$h = -x^2 + 5 - \frac{1}{4}x^2$

$h = -\frac{5}{4}x^2 + 5$

$\Rightarrow V_i = 2\pi(3 - x_i)\left(\frac{-5}{4}x_i^2 + 5\right)\Delta x$

$$\boxed{\text{Shells: } \int_{-2}^{2} 2\pi(3-x)\left(\frac{-5}{4}x^2+5\right)dx}$$

$\frac{1}{4}x^2 = -x^2 + 5$

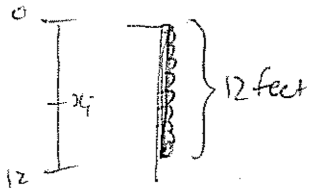
$\frac{5}{4}x^2 = 5$

$x^2 = 4$

$x = \pm 2$

2. (10 points) A chain has length of 12 feet, and it is hanging out of a window. Supposing the chain weighs 5 lbs/ft, how much work is needed to pull 6 feet of the chain into the window? You must use calculus to receive full points.

$$\text{Force}_i = \Delta x \text{ ft} \cdot 5 \text{ lbs/ft} = 5 \Delta x \text{ lbs}$$



$$W_i = \underbrace{5 \Delta x \text{ lbs}}_{F} \cdot \underbrace{x_i \text{ feet}}_{d} = 5x_i \Delta x$$

$$\text{Work} = \int_0^6 5x\,dx + 6 \cdot 5 \cdot 6$$

feet it gets lifted

lbs/ft

feet of rope at bottom

$$= \frac{5}{2}x^2 \Big|_0^6 + 180$$

$$= 90 + 180 = \boxed{270} \text{ lb-feet}$$

Integrate by hand Allow calculator round to nearest 0.01

6. (12 points) ~~NO CALCULATOR~~. Integrate $\int_0^{\sqrt{2}} \frac{x^3}{\sqrt{4-x^2}}\,dx$.

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$0 \to 0 = 2\sin\theta \Rightarrow \theta = 0$

$\sqrt{2} \to \frac{\sqrt{2}}{2} = \sin\theta \Rightarrow \theta = \pi/4$

$$= \int_0^{\pi/4} \frac{8\sin^3\theta}{\sqrt{4-4\sin^2\theta}} \cdot 2\cos\theta\, d\theta$$

$$= \int_0^{\pi/4} \frac{8\sin^2\theta}{2\cancel{\cos\theta}} \cdot 2\cancel{\cos\theta}\, d\theta$$

$$= 8\int_0^{\pi/4} \sin^3\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$0 \to 1$

$\pi/4 \to \frac{\sqrt{2}}{2}$

$$= 8\int_0^{\pi/4} \sin\theta(1-\cos^2\theta)\, d\theta$$

$$= -8\int_1^{\sqrt{2}/2} (1-u^2)\, du$$

$$= -8\left(u - \frac{u^3}{3}\right)\Bigg|_1^{\sqrt{2}/2}$$

$$= -8\left[\frac{\cancel{8}\sqrt{2}}{2} - \frac{1}{3}\cdot\frac{2\sqrt{2}}{\cancel{8}4} - \left(1-\frac{1}{3}\right)\right]$$

$$= -8\left[\frac{6\sqrt{2}-\sqrt{2}}{12} - \frac{8}{12}\right]$$

$$= \frac{-\cancel{8}^{2}(5\sqrt{2}-8)}{\cancel{12}\,3} = \frac{-2}{3}(5\sqrt{2}-8) \approx \boxed{0.62}$$

~~40~~ 0 17 min

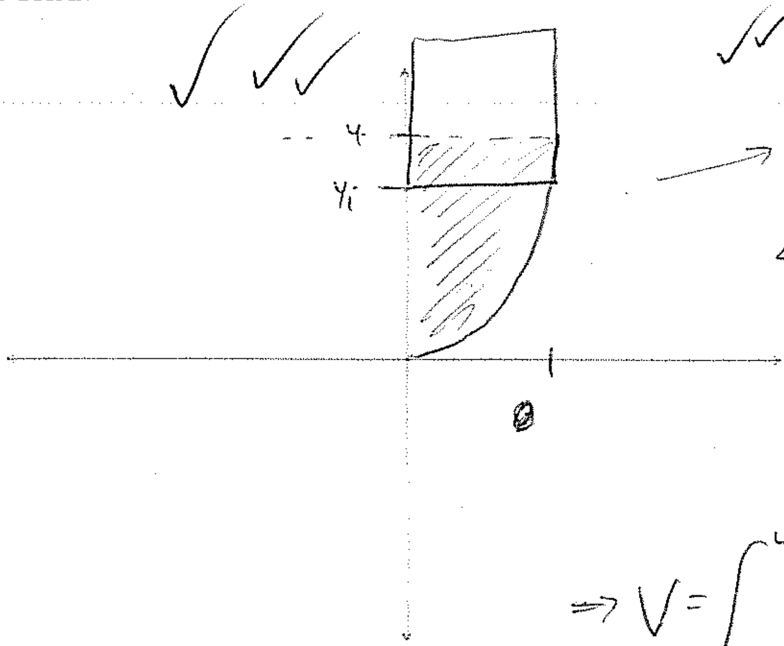
1:52

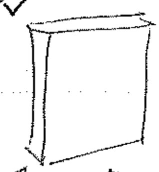
1:56

4 17

$\boxed{21}$

3. (10 points) The base of a solid is the region enclosed by the curves $y = x^4$, $y = 4$, and the y-axis. Cross-sections perpendicular to the y-axis are squares. Find the volume of the solid.





Slice @ y_i

$\leftarrow y_i^{1/4}$

Δy

$x_i = y_i^{1/4}$

$$\Rightarrow V_i = (y_i^{1/4})^2 \Delta y = y_i^{1/2}$$

$$\Rightarrow V = \int_0^4 y^{1/2}\,dy$$

$$= \frac{2}{3} y^{3/2} \Big|_0^4$$

$$= \frac{2}{3} \cdot 8 = \boxed{\frac{16}{3}}$$

4. (10 points) NO CALCULATOR. Integrate $\int_0^3 \frac{1}{(x-2)^3}\,dx$.

Not continuous at $x=2$!

$$= \int_0^2 \frac{1}{(x-2)^3}\,dx + \int_2^3 \frac{1}{(x-2)^3}\,dx$$

$$= \lim_{b\to 2^-} \int_0^b \frac{1}{(x-2)^3}\,dx + \cdots$$

$$= \lim_{b\to 2^-} -\frac{1}{2(x-2)^2}\Bigg|_0^b + \cdots$$

$$= \lim_{b\to 2^-} \frac{-1}{2}\cdot\left(\frac{1}{(b-2)^2} - \frac{1}{(-2)^2}\right) + \cdots$$

goes to ∞ as $b \to 2^-$

$\Rightarrow$ divergent

don't need to do since first part diverges

5. (10 points) NO CALCULATOR. Integrate $\int x^2 \ln(2x)\,dx$.

$u = \ln(2x) \quad dv = x^2\,dx$

$du = \frac{\cancel{2}1}{\cancel{2}x}\,dx \quad v = \frac{x^3}{3}$

✓ ✓ ✓ ✓

$= \ln(2x) \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3}\,dx$ ✓✓

$= \frac{[\ln(2x)]x^3}{3} - \int \frac{x^2}{3}\,dx$ ✓

$=$ " $- \frac{x^3}{9} + C$ ✓✓

$= \boxed{\frac{x^3 \ln(2x)}{3} - \frac{x^3}{9} + C}$ ✓

7. (11 points) NO CALCULATOR. Calculate the arc length of the parametric curve $x = \cos(3t)$, $y = \sin(3t) + 1$ for $0 \leq t \leq 4$.

$$L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = -3\sin(3t)$$

$$\frac{dy}{dt} = 3\cos(3t)$$

$$= \int_0^4 \sqrt{9\sin^2(3t) + 9\cos^2(3t)}\, dt$$

$$= \int_0^4 3\sqrt{\sin^2(3t) + \cos^2(3t)}\, dt$$

$$= \int_0^4 3 \cdot \sqrt{1}\, dt$$

$$= 3t \Big|_0^4 = \boxed{12}$$

Extra Credit(2 points) Find the work needed for question 2 without using calculus. Explain your reasoning.

Weight at beginning = $5 \cdot 12 = 60$ lbs

" " end = $5 \cdot 6 = 30$ lbs

$\Rightarrow$ Average of 45 lbs $\cdot$ 6 feet = $\boxed{270 \text{ ft-lbs}}$