Quiz 6, Calculus 2

2:46
2:49
3

Name: Key

1. (5 points) Use the Maclaurin series for $\cos x$ to obtain the Maclaurin series for

$$x \cos\left(\frac{x^2}{2}\right).$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\Rightarrow \cos\left(\frac{x^2}{2}\right) = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x^2}{2}\right)^{2n} \cdot \frac{1}{(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{4^n (2n)!}$$

$$\Rightarrow x \cos\left(\frac{x^2}{2}\right) = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{4^n (2n)!}}$$

2. (5 points) Find a power series representation for

$$\frac{5x}{(1-x)^2} = 5x\left(\frac{1}{(1-x)^2}\right)$$

and determine the radius of convergence.

$$\frac{1}{(1-x)^2} = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left(\sum_{n=0}^{\infty} x^n\right)$$

$$= \sum_{n=1}^{\infty} n x^{n-1}$$

$$\Rightarrow \frac{5x}{(1-x)^2} = 5x\sum_{n=1}^{\infty} n x^{n-1} = \boxed{\sum_{n=1}^{\infty} 5n\, x^n}$$

converges for $|x|<1$

$$\Rightarrow \boxed{R = 1}$$