Quiz 4, Calculus 2

Name: _____ Key _____

1. (4 points) A trough has the shape of a triangular prism, with the pointy end down. The triangle is isosceles, with a base of 3 feet and height of 6 feet. The trough is 10 feet long. Suppose that the trough is only filled up to <u>half</u> of its depth. Set up, but **do not integrate** an integral that represents how much work is needed to pump out all of the water over the side of the trough. As usual, the density of water is 62.5 lbs/ft³.
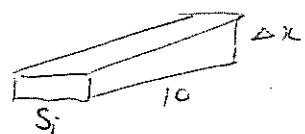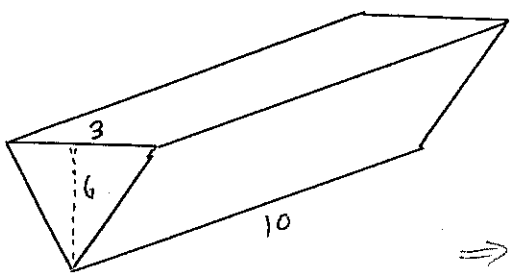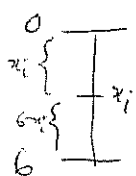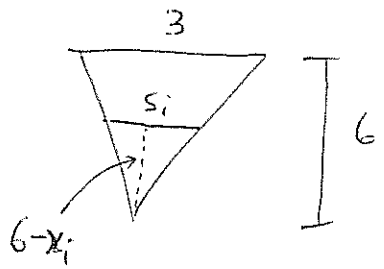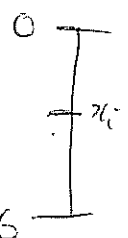

$$V_i = 10 \cdot s_i \cdot \Delta x$$

$$\Rightarrow \cancel{F_i} = 62.5 \cdot 10 \cdot s_i \, \Delta x$$

$$= 625 \cdot s_i \cdot \Delta x$$

$$\Rightarrow Work_i = 625 \cdot s_i \, \Delta x \cdot x_i$$

distance to lift

What is s_i?

$$\Rightarrow Total \; work =$$

$$\int_3^6 625\left(3 - \frac{x}{2}\right) \cdot x \, dx$$

only 3 to 6 because tank is only <u>half</u> full.

so $\dfrac{3}{6} = \dfrac{s_i}{6 - x_i}$

$$\Rightarrow \quad s_i = 3 - \frac{x_i}{2}$$

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2. (2 points) Determine if the sequence $a_n = \dfrac{\cos^2 n}{2^n}$ converges or diverges. If it converges, find the limit. Make sure to explain your reasoning.

$0 \leq \cos^2 n \leq 1$ for all n

$\Rightarrow$ $\dfrac{\cos^2 n}{2^n} \leq \dfrac{1}{2^n}$ and $\left\{\dfrac{1}{2^n}\right\} \to 0$ b/c $2^n \to \infty$

So $\dfrac{\cos^2 n}{2^n}$ converges to 0

3. (4 points) Determine if the series is convergent or divergent. If it converges, find the limit. Make sure to explain your reasoning.

(a) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$ diverges b/c it is the harmonic series.

(b) $\displaystyle\sum_{n=0}^{\infty} \dfrac{3^{n+1}}{\pi^n} = \sum_{n=0}^{\infty} 3 \cdot \left(\dfrac{3}{\pi}\right)^n$

$\uparrow a$ $\uparrow r$

$\dfrac{3}{\pi} < 1$ so it converges

to $\dfrac{a}{1-r} = \dfrac{3}{1 - \frac{3}{\pi}}$

or $\dfrac{3\pi}{\pi - 3}$ or ≈ 66.56

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