Quiz 3, MTH 1420

12:34

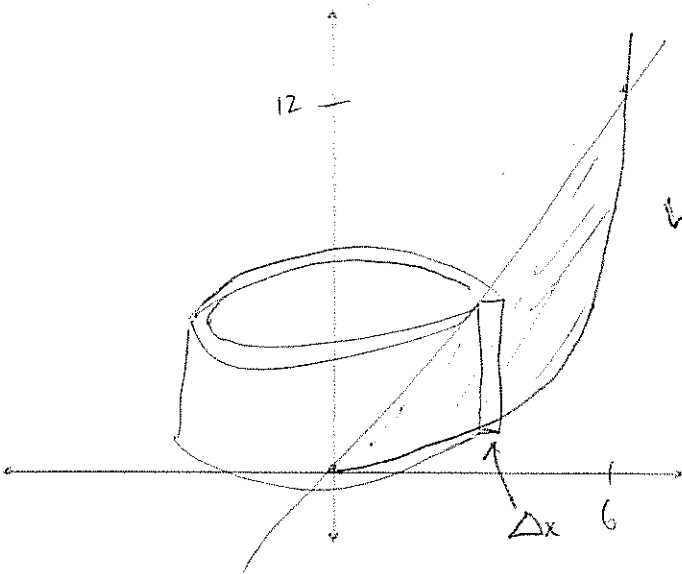
Name: Key

Calculators are allowed on this test.

1) (5 points) Consider the region R enclosed by the curves $y = \frac{x^2}{3}$ and $y = 2x$. Set up but do not evaluate an integral to represent the volume of the solid formed when the region R is rotated about the y-axis. Make a sketch to show how you are making your cross-sections.

$\frac{x^2}{3} = 2x \Rightarrow x^2 - 6x = 0$

$x(x-6) = 0$



$V_{slice} = 2\pi r h \Delta x$

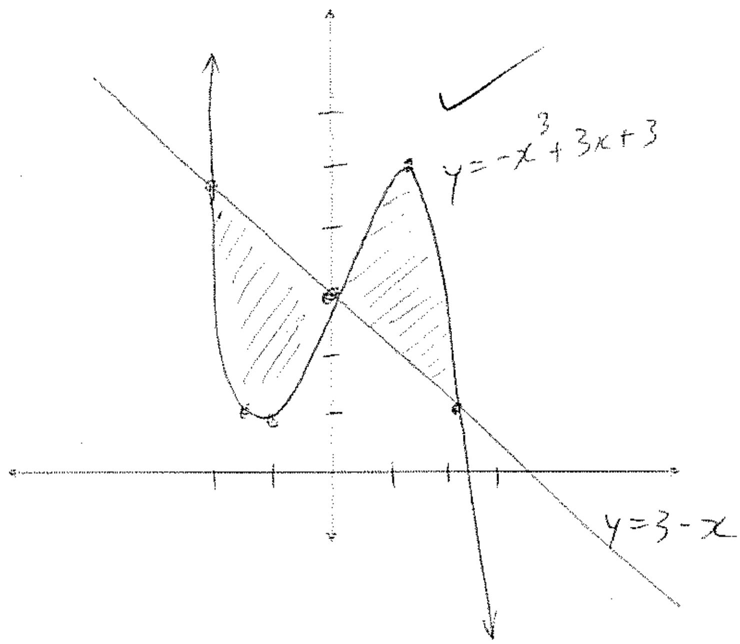
$= 2\pi x(2x - (x^2 - 6x))\Delta x$

Shells: $\int_0^6 2\pi x(8x - x^2)\,dx$

Washers: $\int_0^{12} \pi\left((\sqrt{3y})^2 - (\frac{y}{2})^2\right)dy$

$= \pi \int_0^{12} \left(3y - \frac{y^2}{4}\right) dy$

2) (5 points) Find the area completely enclosed by the curves $y = -x^3 + 3x + 3$ and $y = 3 - x$. Make a sketch to show what you are doing, and also SHOW YOUR WORK to explain how you find your limits of integration (it is not enough to just say you found them on the calculator). You must do the integration by hand, but can check your answer with a calculator.



$-x^3+3x+3=3-x$

$0=x^3-4x$

$0=(x^2-4)x$

$0=(x-2)(x+2)x$

$\Rightarrow$ intersections at $2, -2, 0$

$$\text{Area}=\int_{-2}^{0}\left[(3-x)-(-x^3+3x+3)\right]dx+\int_{0}^{2}\left[(-x^3+3x+3)-(3-x)\right]dx$$

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$$=\int_{-2}^{0}(x^3-4x)\,dx+\int_{0}^{2}(-x^3+4x)\,dx$$

$$=\tfrac{1}{4}x^4-2x^2\Big|_{-2}^{0}+-\tfrac{1}{4}x^4+2x^2\Big|_{0}^{2}$$

$$=0-(4-8)+(-4+8)$$

$$=4+4=\boxed{8}$$

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