Calculus III, Test 3 Review Answers

Dr. Graham-Squire, Fall 2013

•The test will cover sections 13.9, 13.10, and 14.1-14.8.

1. The paint for a room costs \$0.05 per square foot for ceiling paint and \$0.12 for paint for the walls (no paint on the floor). If the room has volume of 1000 ft^3, what are the dimensions for the room that will minimize the cost of the paint? What will that minimum cost be?

   Ans: 16.87 by 16.87 by 3.51. The minimum cost will be \$42.68.

2. Find the maximum production level P if the total cost of labor (at \$72 per unit) and capital (at \$60 per unit) is limited to \$250,000, where x is the number of units of labor and y is the number of units of capital and $P(x, y) = 100x^{0.4}y^{0.6}$.

   Ans: 197,620.

3. Evaluate the iterated integral
$$\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{2+y^3}\,dy\,dx.$$

   Ans: $\ln 5$

4. Find the volume of the solid formed by the intersection of the surfaces $z = 1 - xy$, $y = 1$, $y = x$, $z = 0$, and $x = 0$. You can use Sage/Maple/Grapher to help you figure out what it looks like if you need to.

   Ans: 3/8

5. Evaluate the integral:
$$\int_0^3 \int_0^{\sqrt{9-x^2}} (x^2+y^2)^{3/2}\,dy\,dx$$

   Ans: $243\pi/10$

6. Find the center of mass of the lamina bounded by $y = 4 - x^2$ and $y = 0$, with a density function of $\rho = ky$.

   Ans: $\bar{x} = 0$ by symmetry. Then $m = 17.07k$, $M_x = 4096k/105$, and the center of mass is $(0, 2.29)$.

7. Write a double integral that represents the surface area of $f(x, y) = x^2 + y^2$ over the region $R : \{(x, y) : 0 \leq f(x, y) \leq 16\}$. Integrate by hand if you can, and if you cannot explain why and find the integral using a computer algebra system (like Sage/Maple).

   Ans: $\displaystyle\int_0^{2\pi} \int_0^4 r\sqrt{1+4r^2}\,dr\,d\theta = \frac{(65\sqrt{65}-1)\pi}{6}$

8. Sketch the solid whose volume is given by the iterated integral $\displaystyle\int_0^4 \int_0^{(4-x)/2} \int_0^{(12-3x-6y)/4} dz\,dy\,dx.$ Then rewrite the integral to have order of integration $dz\,dx\,dy$, and compute whichever integral you think is easier.

   Ans: $\displaystyle\int_0^2 \int_0^{4-2y} \int_0^{(12-3x-6y)/4} dz\,dx\,dy = 4$

9. For the triple integral

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dy\, dx,$$

convert from rectangular to both cylindrical and spherical coordinates. If one conversion is particularly annoying/nasty, you can also just explain why it is bad. Choose the easiest method and do the integration.

Ans: $\displaystyle\int_0^{\pi/2} \int_0^2 \int_0^{\sqrt{16-r^2}} r^2\, dz\, dr\, d\theta = \frac{8\pi^2}{3} - 2\pi\sqrt{3}$ in cylindrical

In spherical: $\displaystyle\int_0^{\pi/2} \int_0^{\pi/6} \int_0^4 \rho^3 \sin^2\phi\, d\rho\, d\phi\, d\theta + \int_0^{\pi/2} \int_{\pi/6}^{\pi/2} \int_4^{2\csc\phi} \rho^3 \sin^2\phi\, d\rho\, d\phi\, d\theta = \frac{8\pi^2}{3} -$ $2\pi\sqrt{3}$

For the cylindrical one, you could also skip the calculation and just point out that the conversion is difficult because ρ has to run between two different functions- first it is being calculated up to the portion of the sphere, then it is calculated up to the cylinder. This is why we have to do two different integrals and add them together.

10. Use a change of variables to find the solid region lying below the surface

$$f(x, y) = (x+y)^2 \sin^2(x-y)$$

and above the region R which is bounded by the square with vertices $(\pi, 0)$, $(3\pi/2, \pi/2)$, (π, π), $(\pi/2, \pi/2)$.

Ans: $\dfrac{7\pi^4}{12}$