# Calculus III, Test 2 Review

Dr. Graham-Squire, Fall 2013

•The test will cover sections 12.1 through 13.8.

•To study, you can look over your notes, rework HW problems on WebAssign, quizzes, and problems from the notes, as well as work out the practice problems given for each section. The Review Questions at the end of Chapters 12 and 13 are also good practice, as are the materials from last year which you can find on my website.

•Calculators are allowed on this test, but for certain questions you may not be allowed to use a calculator. There will also be a question or two where you will be able to use a computer (though a computer may not be necessary). •Some practice problems to work on:

1. Calculate the limits:

   (a) $\lim_{t\to 0}\left(t^2\mathbf{i}+\frac{1-\cos t}{t}\mathbf{j}+\frac{3}{\ln t}\mathbf{k}\right)$

   (b) $\lim_{(x,y)\to(0,0)}\frac{2x-y^2}{2x^2+y}$

2. Evaluate the definite integral by hand: $\int_0^2 (t\mathbf{i}+te^{t^2}\mathbf{j}-te^t\mathbf{k})\,dt$. You should leave your answer in exact form (no decimal approximation).

3. A baseball player at second base throws a ball 90 feet to the player at first base. The ball is released at a point 5 feet above the ground with an initial velocity of 50 mph and an angle of 15° above horizontal. At what height does the player at first base catch the ball?

4. Let $\mathbf{r}(t)=\sqrt{2}t\mathbf{i}+e^t\mathbf{j}+e^{-t}\mathbf{k}$. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ at $t=0$. Hint: simplify $||\mathbf{r}'(t)||$ before you find $\mathbf{N}$.

5. For $\mathbf{r}(t)=\sqrt{2}t\mathbf{i}+e^t\mathbf{j}+e^{-t}\mathbf{k}$ as in the previous problem, find the curvature $K$.

6. For $f(x,y)=\ln(xy-6)$, do the following:

   (a) Describe the domain and range.

   (b) Sketch level curves at $c=-10, c=0$, and $c=2$ (you can use a graphing calculator and/or Sage to help with this).

   (c) Find $\nabla f(x,y)$.

   (d) On the sketch from part (b), plot $\nabla f(1,7)$ as a vector with its initial point at (1,7). What do you (or should you) notice?

7. Discuss the continuity of the function $f(x,y)=-\frac{xy^2}{x^2+y^4}$. Where will it be continuous and where will it be discontinuous (if any)? Back up your assertions with mathematical reasoning. Graph $f$ and explain how the graph reinforces your assertions.

8. The formula for wind chill is given by

$$C = 35 + 0.6T - 36v^{0.16} + 0.4Tv^{0.16}$$

where $v$ is wind speed (in mph) and $T$ is temperature in Fahrenheit. The wind speed is $23 \pm 3$ mph and the temperature is $8^\circ \pm 1^\circ$. Calculate $C$ at $(v, t) = (23, 8)$ and use differentials to estimate the maximum propagated error for the given situation.

9. Use partial derivatives to differentiate implicitly to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for the equation

$$x + \sin(y + z) = 0.$$

10. Let $g(x, y) = 2xe^{y/x}$.

    (a) Find the direction of maximum increase at $(x, y)$=(2,0).

    (b) Find the slope if you were walking from (2,0) in the direction of the point (5,3).

11. Find the point(s) on the surface $z = 3x^2 + 2y^2 - 3x + 4y - 5$ at which the tangent plane is horizontal.

12. Find the absolute extrema of $f(x, y) = 3x^2 + 2y^2 - 4y$ over the region in the $xy$-plane bounded by the graph of $y = x^2$ and $y = 4$.