Minitest 4 - MTH 2410

Dr. Graham-Squire, Fall 2013

12:01

12:23

⇒ give 40 minutes

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

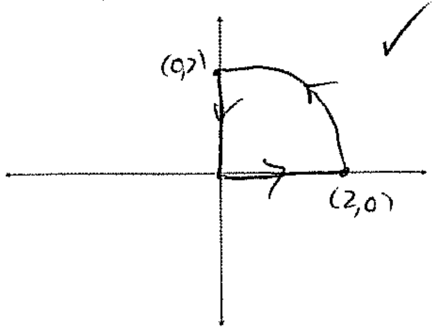
DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Calculators are allowed on all parts of the in-class portion of the test, though you should not need one.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 6. Total Points = 30.

For the first four questions, evaluate the line integral using whatever technique you feel is most appropriate.

$M \quad N$

1. (5 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle xy, y^2 \rangle$ and C is the curve from the origin to (2,0) in a straight line, then from (2,0) to (0,2) along a semicircular path of radius 2, then from (0,2) back to the origin in a straight line.



is a loop. Use Green's Thm:

$$\int_C F \cdot dr = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \, dA$$

$$\frac{\partial N}{\partial x} = 0, \quad \frac{\partial M}{\partial y} = x$$

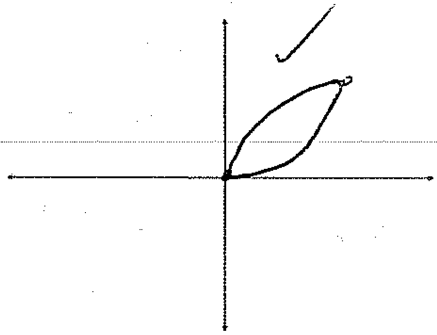
do polar: $0 \leq r \leq 2$, $0 \leq \theta \leq \frac{\pi}{2}$

$$\int_0^{\pi/2} \int_0^2 -r\cos\theta \cdot r \, dr \, d\theta$$

$$= -\int_0^{\pi/2} \left(\frac{1}{3} r^3 \Big|_0^2 \right) \cos\theta$$

$$= -\frac{8}{3} \left(\sin\theta \Big|_0^{\pi/2} \right) = -\frac{8}{3}(1-0) = \boxed{-\frac{8}{3}}$$

2. (5 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle 2x + \tan y, x \sec^2 y \rangle$ and C is the curve from the origin to (1,1) along the graph of $y = x^2$, then from (1,1) back to (0,0) along the graph of $y = \sqrt{x}$.



loop. Use Green's theorem

$$\int_C F \cdot d_r = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \, dA$$

$$\frac{\partial N}{\partial x} = \sec^2 y \qquad \frac{\partial M}{\partial y} = \sec^2 y \quad \Rightarrow \quad \text{Conservative!}$$

$$\iint_R (\sec^2 y - \sec^2 y)\, dA = \iint_R 0 \, dA = \boxed{0}$$

3. (5 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle xy, yz, zx \rangle$ and C is the curve given by $x = t$, $y = t^2$, $z = t^3$ from the point (0,0,0) to (2, 4, 8).

$r(t) = \langle t, t^2, t^3 \rangle \quad 0 \le t \le 2$

$r'(t) = \langle 1, 2t, 3t^2 \rangle dt$

F conservative?

$\int xy\, dx = \frac{1}{2}x^2 y + C(y, $

$\int yz\, dy = \frac{1}{2}y^2 z + C(x, z)$

$\int zx\, dz = \frac{1}{2}z^2 x + C(x, y)$

does not work.

$F \cdot dr = \langle t^3, t^5, t^4 \rangle \cdot \langle 1, 2t, 3t^2 \rangle dt$

$= (t^3 + 2t^6 + 3t^6)\, dt$

$\Rightarrow \int_0^2 (t^3 + 2t^6 + 3t^6)\, dt$

$= \int_0^2 (t^3 + 5t^6)\, dt$

$= \frac{1}{4}t^4 + \frac{5}{7}t^7 \Big|_0^2 = 4 + \frac{128(5)}{7} = \frac{28 + 640}{7} = \boxed{\frac{668}{7}}$

$= 4 + \frac{640}{7}$

4. (5 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$ and C is the curve given by $x = t^2$, $y = \sqrt{t}$, $z = t/2$ from $t = 0$ to $t = 4$.

$\Rightarrow F$ conservative ✓

$\int y^2 dx = xy^2 + C$

$F = \nabla f$ for $f = xy^2 + ye^{3z}$ ✓ ✓

$\int (2xy + e^{3z}) dy = xy^2 + ye^{3z} +$

$t=0 \Rightarrow (0,0,0)$

$\int 3ye^{3z} dz = ye^{3z} + C$

$t=4 \Rightarrow (16, 2, 2)$ ✓

$$\int F \cdot dr = xy^2 + ye^{3z} \Big|_{(0,0,0)}^{(16,2,2)}$$ ✓

$= 16 \cdot 4 + 2e^6$

$= \boxed{64 + 2e^6}$

5. (5 points) (a) Set up, but <u>do not evaluate</u> an iterated integral to calculate the surface integral $\iint_S x^2 z^2 dS$ where S is the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 3$.

$z = \sqrt{x^2+y^2} = g(x,y)$

(b) Is it easy to compute the integral directly? Explain why or why not. If it is easy, calculate it. If not, explain what you could do to simplify it.

(a) $\iint_S x^2 z^2 dS = \iint_R x^2(x^2+y^2)\sqrt{1+g_x^2+g_y^2}\,dA$ ✓

$g_x = \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2x$ ✓

$g_x = \frac{x}{\sqrt{x^2+y^2}}$

$g_y = \frac{y}{\sqrt{x^2+y^2}}$

$= \iint_R x^2(x^2+y^2)\sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}\,dA$

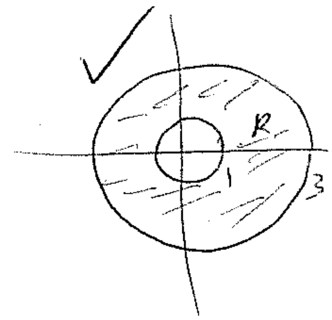
$z=1$

$\Rightarrow 1 = x^2+y^2$

$= \iint_R x^2(x^2+y^2)\sqrt{2}\,dA$ ✓

$z=3$

$\Rightarrow 9 = x^2+y^2$



region hard to write in rectangular.

do in polar: $1 \le r \le 3$

$0 \le \theta \le 2\pi$

$\int_0^{2\pi}\int_1^3 r^2\cos^2\theta(r^2)\sqrt{2}\cdot r\,dr\,d\theta$ ✓

$$\boxed{= \int_0^{2\pi}\int_1^3 r^5\cos^2\theta\,dr\,d\theta}$$

$= \int_0^{2\pi} \frac{r^6}{6}\Big|_1^3 \cos^2\theta\,d\theta$

$= \left(\frac{3^6}{6}-\frac{1}{6}\right)\int_0^{2\pi}\cos^2\theta\,d\theta$

$= \left(\frac{3^6-1}{6}\right)\int_0^{2\pi}\left(\frac{1}{2}+\frac{\cos 2\theta}{2}\right)d\theta = \left(\frac{3^6-1}{6}\right)\left(\frac{1}{2}\theta+\frac{\sin(2\theta)}{4}\Big|_0^{2\pi}\right)$

$= \boxed{\left(\frac{3^6-1}{6}\right)\pi}$

6. (5 points) Use an iterated integral to find the surface area of the sphere of radius q, given parametrically by

$$\mathbf{r}(u,v) = q\sin u\cos v\mathbf{i} + q\sin u\sin v\mathbf{j} + q\cos u\mathbf{k}$$

where the domain D is $0 \le u \le \pi$ and $0 \le v \le 2\pi$.

$$S.A. = \int_0^{\pi}\int_0^{2\pi} \|r_u \times r_v\| \, dv\, du \quad 0.5$$

$$r_u = \langle q\cos u\cos v, q\cos u\sin v, -q\sin u \rangle$$

$$0.5 \quad r_v = \langle -q\sin u\sin v, q\sin u\cos v, 0 \rangle$$

$$= \int_0^{\pi}\int_0^{2\pi} q^2 \sin u \, dv\, du$$

$$\begin{vmatrix} i & j & k \\ q\cos u\cos v & q\cos u\sin v & -q\sin u \\ -q\sin u\sin v & q\sin u\cos v & 0 \end{vmatrix}$$

$$= 2\pi q^2 \left. -\cos u \right|_0^{\pi}$$

$$= 2\pi q^2(-(-1)-(-1))$$

$$= \boxed{4\pi q^2}$$

$$r_u \times r_v = \langle q^2\sin^2 u\cos v, q^2\sin^2 u\sin v, q^2\cos u\sin u\cos^2 v + q^2\cos u\sin u\sin^2 v \rangle$$

$$r_u \times r_v = q^2 \langle \sin^2 u\cos v, \sin^2 u\sin v, \cos u\sin u \rangle$$

$$\|r_u \times r_v\| = q^2\sqrt{\sin^4 u\cos^2 v + \sin^4 u\sin^2 v + \cos^2 u\sin^2 u}$$

$$= q^2\sqrt{\sin^2 u(\sin^2 u + \cos^2 u)}$$

$$= q^2 \sin u$$

Extra Credit(1 to 3 points) Choose whether you want to get 1 or 3 extra credit points. If you choose 1, you are guaranteed to get 1 point. If you choose 3 and no one else chooses 3, you get 3 points. But if anyone else in the class also chooses 3, everyone who chooses 3 will get zero.