Calculus III, MiniTest 4 Review

Dr. Graham-Squire, Fall 2013

1. Find the curl of the vector field $\mathbf{F}(x, y) = \dfrac{yz\mathbf{i} - xz\mathbf{j} - xy\mathbf{k}}{y^2z^2}$. Is $\mathbf{F}$ conservative? If so, find f such that $\nabla f = \mathbf{F}$.

   Ans: curl $\mathbf{F}$=0, so $\mathbf{F}$ is conservative. $f(x, y, z) = \dfrac{x}{yz}$ is a function such that $\nabla f = \mathbf{F}$.

2. Evaluate the line integrals:

   (a) $\int_C (2x - y)dx + (x + 2y)dy$ where C is given by:

   (i) C: one revolution counterclockwise around the circle $x = 3\cos t$, $y = 3\sin t$.

   Ans: Can use Green's theorem to get 18π.

   (ii) C: the line segment from (0,0) to (3,-3).

   Ans: Have to calculate the line integral directly to get 18.

   (b) $\int_C xy\,dx + \frac{1}{2}x^2\,dy$, where C is the boundary of the region between the graphs of $y = x^2$ and $y = 1$.

   Ans: Can use either the fundamental theorem of line integrals or Green's thm. to get 0.

   (c) $\int y\,dx + x\,dy + \frac{1}{z}\,dz$ where C is the curve $\mathbf{r}(t) = \langle t, t^2 - 3t, \frac{3}{4}t + 1\rangle$, $0 \le t \le 4$.

   Ans: Use Fund. theorem to get 16+ln 4.

   (d) $\int_C (x^2 - y^2)\,dx + 2xy\,dy$, where C is given by $x^2 + y^2 = a^2$ (a is some constant).

   Ans: Use Green's theorem to get 0.

   (e) $\int_C xy\,ds$ where C is the line segment from (0,0) to (5,4).

   Ans: Need to use formula for line integrals (the one that involves the arc length) to get $\frac{20\sqrt{41}}{3}$.

3. Find an equation for the tangent plane to the paraboloid given by

$$\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + (u^2 + v^2)\mathbf{k}$$

   at the point (1,2,5).

   Ans: $-2x - 4y + z = -5$

4. Evaluate the surface integral $\iint_S z\, dS$ over the surface given by

$$\mathbf{r}(u, v) = (u + v)\mathbf{i} + (u - v)\mathbf{j} + \sin v\mathbf{k}$$

where $0 \leq u \leq 2$ and $0 \leq v \leq \pi$. You may have to use Sage/Maple to evaluate the integral.

Ans: $\displaystyle\int_0^\pi \int_0^2 \sin v\sqrt{2\cos^2 v + 4}\, du\, dv = 8.623.$