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Quiz 5, Calculus III – Computers needed

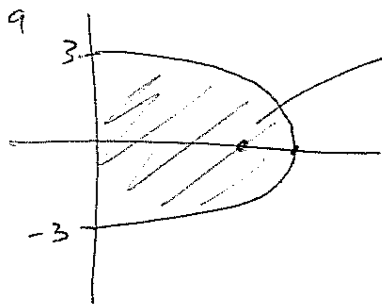
Dr. Graham-Squire, Fall 2013

Name: Key

1. (5 points) Let L be the lamina bounded by the graphs $x = 9 - y^2$ and $x = 0$, with density function given by $\rho(x, y) = x^2$.

   (a) Sketch a graph of the region. Given your knowledge of the shape of the lamina and the density function, estimate where the center of mass $(\bar{x}, \bar{y})$ will lie. You can write you answer in coordinates or just place a dot somewhere on the lamina. Make sure to explain your answer.

   (b) Set up integrals to find the unknown values from part (a). Use Sage/Maple to calculate the center of mass to see how close your estimate was from part (a).



$\bar{y} = 0$ b/c it is symmetric w/respect to x-axis and $\rho(x,y) = x^2 \Rightarrow$ heavier for higher x-values.

$\Rightarrow$ guess $(7, 0) = (\bar{x}, \bar{y})$

(b) $\bar{x} = \frac{M_y}{m}$

$$m = \int_{-3}^{3} \int_{0}^{9-y^2} x^2 \, dx \, dy = 2332^{8}/_{25}$$

$$M_y = \int_{-3}^{3} \int_{0}^{9-y^2} x^3 \, dx \, dy = 13996^{8}/_{35}$$

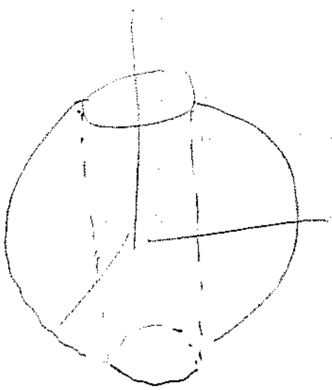
$\boxed{\frac{M_y}{m} = 6} \Rightarrow (6, 0)$ is center of mass.

2. (5 points) (a) Set up (but do not evaluate) a triple integral to determine the volume of the 3-dimensional region lying inside the sphere $x^2+y^2+z^2=4$ but outside the cylinder $x^2+y^2=1$. You can set up the integral in whatever coordinate system you see fit.

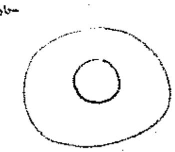
(b) Will you be able to integrate your triple integral by hand? Explain why or why not.

(c) Use Sage/Maple to evaluate the integral. Does your answer make sense? $\Rightarrow$ Note that Volume of a sphere is $\frac{4}{3}\pi r^3$ cylinder is $V = \pi r^2 h$

or do it by hand



Polar Region of integration



$\Rightarrow$ $1 \le r \le 2$

$0 \le \theta \le 2\pi$

$-\sqrt{4-x^2-y^2} \le z \le \sqrt{4-x^2-y^2}$

$-\sqrt{4-r^2} \le z \le \sqrt{4-r^2}$

$2\sqrt{3}\pi$

(a) $$\boxed{V = 2\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta}$$

(b) Yes. when I integrate the dz, get $r\sqrt{4-r^2}$ which can be done with substitution.

(c) $$V = 4\pi \int_1^2 r z \Big|_0^{\sqrt{4-r^2}} dr$$

$$= 4\pi \int_1^2 r\sqrt{4-r^2}\,dr$$

$u = 4 - r^2$

$du = -2r\,dr$

$-\frac{1}{2}\,du = r\,dr$

r	u
2	0
1	3

$$= -2\pi \int_3^0 u^{1/2}\,du$$

$$= -2\pi \, \frac{2}{3} u^{3/2}\Big|_3^0$$

$$= -2\pi\left(-\frac{2}{3}\cdot 3^{3/2}\right) = 4\pi\sqrt{3} = 21.77$$

Volume of sphere = $\frac{4}{3}\pi(2^3) = \frac{32}{3}$