Quiz 3, Calculus III – ~~No Calculators~~

Dr. Graham-Squire, Fall 2013

9:35
9:41
6 min
$\Rightarrow$ 24 min.

Name: Key

1. (3 points) Calculate the limit: $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

check $x=0 \Rightarrow \lim_{y\to 0} \frac{0}{0+y^2} = \lim_{y\to 0} \frac{0}{y^2} = 0$

$y=0 \Rightarrow \lim_{x\to 0} \frac{0}{x^2+\cancel{0}^2} = \lim_{x\to 0} \frac{0}{x^2} = 0$

$x=y \Rightarrow \lim_{x\to 0} \frac{x^2}{x^2+x^2} = \lim_{x\to 0} \frac{1\cancel{x^2}}{2\cancel{x^2}} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$

$\Rightarrow$ limit dne

2. (3 points) The electrical power P is given by $P = \frac{E^2}{R}$ where E is voltage and R is resistance. Use differentials to approximate the maximum percentage error in calculating the power when 120 volts (with error of ± 4 volts) is applied to a 2000 ohm resistor (with an error of ± 80 ohms).

$dP = P_E\, dE + P_R\, dR$

$P_E = \frac{2E}{R}$ $\quad P_R = \frac{-E^2}{R^2}$ $\quad dE = \pm 4, \quad dR = \pm 80$

$P_E(120, 2000) = \frac{2(120)}{2000}$ $\quad P_R(120, 2000) = \frac{-(120)^2}{2000^2}$

+4 $\quad$ −80 $\quad$ choose

$dP = \frac{2(120)}{2000} \cdot (\pm 4) + \frac{-(120^2)}{2000^2} \cdot (\pm 80)$

$= \frac{480}{1000} + \frac{(120^2)(80)}{(2000)^2} = 0.48 + 0.288 = \boxed{0.768}$

3. (4 points) Eva is climbing a glacier which has the shape given by the function

$$f(x,y) = 10 - 2x^2 + 6y^2 + 3xy - 5x + 2y$$

and Eva is currently at the point (0,0).

(a) If she spots her teddy bear at the location with (x, y)-coordinates of (3,4) and decides to walk directly towards it, what will be the slope in that direction?

(b) If she instead decides to move from (0,0) and go in the direction of the fastest possible descent, in what direction should she walk? Give your answer in the form of a vector.

$f_x = -4x + 3y - 5$

$f_y = 12y + 3x + 2$

$\nabla f(0,0) = \langle -5, 2 \rangle$

direction to $(3,4)$ is $\langle 3,4 \rangle$ $\quad \|\langle 3,4\rangle\| = \sqrt{3^2+4^2} = 5$

$\Rightarrow \vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

(a) $\nabla f \cdot \vec{u} = -5\left(\frac{3}{5}\right) + 2\left(\frac{4}{5}\right) = -3 + \frac{8}{5} = \boxed{\frac{-7}{5}}$

(b) Fastest descent is the direction of <u>negative</u> gradient

$\Rightarrow -\nabla f = \boxed{\langle 5, -2 \rangle}$