Quiz 6, Calculus III – Calculators okay

Dr. Graham-Squire, Fall 2013

Name: Key

8:23
8:30
7 ⇒ 20 min

Evaluate the following line integrals. You can use any method you choose (direct, fundamental theorem of line integrals, Green's theorem, etc) as long as that method is applicable.

1. (4 points) Calculate

$$\int_C \big(\cos(zy) - \sin(zy)\big)dx + \big(-xz\sin(zy) - xz\cos(zy)\big)dy + \big(-xy\sin(zy) - xy\cos(zy)\big)dz$$

where C is the straight line from the origin to the point $(-2, 1, \frac{\pi}{2})$.

$$\int [\cos(zy) - \sin(zy)]\,dx = x\big(\cos(zy) - \sin(zy)\big) = x\cos(zy) - x\sin(zy) = f(x,y,z)$$

$$\left(\frac{\partial}{\partial y}\right)\big(x\cos(zy) - x\sin(zy)\big) = -xz\sin(zy) - xz\cos(zy)$$

$\Rightarrow F$ is conservative ✓

$$\Rightarrow \int_C F\cdot dr = x\cos(zy) - x\sin(zy)\Big|_{(0,0,0)}^{(-2,1,\frac{\pi}{2})}$$

$$= -2\cos\left(\tfrac{\pi}{2}\right) + 2\sin\left(\tfrac{\pi}{2}\right) - (0)$$

$$= 0 + 2 = \boxed{2}$$

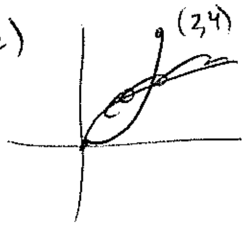
2. (3 points) Evaluate the line integral of $f(x, y) = 8x$ over the curve $\mathbf{r}(t) = \langle t, t^2 \rangle$ in the xy-plane, from the point (0,0) to the point (2,4).

3. (3 points) Evaluate $\int_C F \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle xy, xy^3 \rangle$ and C is the boundary of the rectangle given by the intersection of the lines $y = 0$, $y = 2$, $x = 0$ and $x = 3$.

$x'(t) = 1$

$y'(t) = 2t$

(2)

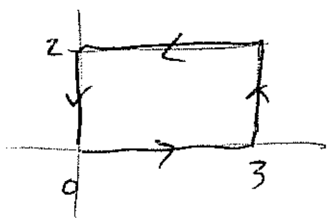


$$\int_0^2 8x \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

$$= \int_0^2 8t \sqrt{1 + 4t^2}\, dt$$

$$= \frac{2}{3}(1+4t^2)^{3/2} \Big|_0^2 = \boxed{\frac{2}{3}\left(17^{3/2} - 1\right)}$$

(3) $\frac{\partial}{\partial y}(xy) = x$ $\quad \frac{\partial}{\partial x}(xy^3) = y^3 \Rightarrow$ not conservative.



is a loop $\Rightarrow$ Use Green's Theorem.

$M = xy \quad , \quad N = xy^3$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$= \int_0^2 \int_0^3 (y^3 - x)\, dx\, dy = \int_0^2 \left(y^3 x - \frac{1}{2}x^2 \Big|_0^3 \right) dy$$

$$= \int_0^2 \left(3y^3 - \frac{9}{2}\right) dy$$

$$= \frac{3}{4}y^4 - \frac{9}{2}y \Big|_0^2 = 12 - 9 - 0$$

$$= \boxed{3}$$