Quiz 1, Calculus III

Dr. Graham-Squire, Fall 2013

10:05
10:11
6 min

Name: Key

1. (3 points)

   State if each of the following are True or False. In either case, give a brief explanation for why (using either words or a calculation).

   (a) If $\mathbf{u}$ is orthogonal to $\mathbf{w}$, and $\mathbf{v}$ is orthogonal to $\mathbf{w}$, is $\mathbf{u}+\mathbf{v}$ orthogonal to $\mathbf{w}$?

   (b) If $\mathbf{u}$ and $\mathbf{v}$ are two nonzero vectors that are not parallel, then $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

(a) Yes (True), $\vec{u}+\vec{v} \perp \vec{w}$. $\vec{u} \perp \vec{w} \Rightarrow \vec{u}\cdot\vec{w} = 0$ ✓✓

$\vec{v} \perp \vec{w} \Rightarrow \vec{v}\cdot\vec{w} = 0$

So $(\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w} = 0+0 = 0 \Rightarrow \boxed{(\vec{u}+\vec{v}) \perp \vec{w}}$

(b) False. In general if $\vec{u} \neq \vec{0} \neq \vec{v}$ and $\vec{u} \neq k\vec{v}$, then $\vec{u}\times\vec{v} = -\vec{v}\times\vec{u}$. ✓✓

2. (3 points)

   For each of the following, state if the expression gives a vector, a scalar, or does not exist. Assume that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all nonzero vectors, and c is a scalar.

lose 0.5 for each wrong

   (i) $(\mathbf{u}\cdot\mathbf{v})\mathbf{w}$ vector

   (ii) $c(\mathbf{u}\times\mathbf{v})$ vector

   (iii) $(\mathbf{u}\cdot\mathbf{v})\times\mathbf{w}$ DNE (cannot cross a scalar)

   (iv) $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}$ scalar

   (v) $c\times(\mathbf{u}\times\mathbf{v})$ DNE (cannot cross a scalar)

3. (4 points) The following lines in space intersect at a certain point.

ℓ_1 $\quad x = 1 + 2t, y = 2, z = 5 - 3t \qquad$ and $\qquad x = 2 - t, y = -2 - 4t, z = 3 + t$ $\quad \ell_2$

Find the direction vectors of each line, and use them to answer the following:

(a) What is the angle between the two lines?

(b) There is a plane that is defined by the two lines. Find a vector normal to that plane.

ℓ_1 has direction vector $\langle 2, 0, -3 \rangle = \vec{u}$ ✓

ℓ_2 " " " $\langle -1, -4, 1 \rangle = \vec{v}$ ✓

10:09
4

(a) angle between θ is ✓ $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$ ✓

10:21
10:23
2

$$\cos\theta = \frac{-2 + 0 - 3}{(\sqrt{13})(\sqrt{18})}$$

$$\theta = \cos^{-1}\left(\frac{-5}{(\sqrt{13})(\sqrt{18})}\right) \approx 109^\circ \text{ or } 1.9 \text{ rad.}$$ ✓

(b) $\ell_1 \times \ell_2 =$

$$\begin{matrix} i & j & k & i & j \\ 2 & 0 & -3 & 2 & 0 \\ -1 & -4 & 1 & -1 & -4 \end{matrix}$$ ✓

$$= (0 - (12))\vec{i} + (+3 - 2)\vec{j} + (-8 - 0)\vec{k}$$ ✓

$$= \langle 12, 1, -8 \rangle$$ (or any scalar multiple)

✓