

Test 2 - MTH 2410  
Dr. Graham-Squire, Fall 2012

Name: \_\_\_\_\_

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(signature)

## DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Computers are allowed on one part of this test. Calculators are allowed on all parts of the test except for the last question, for which no technology is allowed. Even on questions where technology is allowed, you should still show all of your work.
4. Give all answers in exact form, not decimal form (that is, put  $\pi$  instead of 3.1415,  $\sqrt{2}$  instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 11. Total Points = 100.

**Computer allowed for this question**

1. (12 points) Find the absolute maximum and absolute minimum values for

$$f(x, y) = x^3 - 3xy + y^3$$

in the square region given by  $R = \{(x, y) : 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 2\}$ . You can use Sage/Maple/Grapher/Wolfram Alpha to help you figure out what the answers should be, but all answers need to be supported by correct mathematics.

Calculators allowed

Name: \_\_\_\_\_

2. (8 points) Let  $\mathbf{a}(t) = -\cos t \mathbf{i} - \sin t \mathbf{j}$  be an acceleration function. Find the corresponding velocity and position functions given the initial conditions that

$$\mathbf{v}(0) = \mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{r}(0) = \mathbf{i}.$$

3. (8 points) TRUE OR FALSE. Circle the correct answer. If false, give a counterexample or explain (briefly) why it is false. If true, no explanation is necessary (though if you are wrong, an explanation can get you some partial credit).

(a) **True or False:** If  $f$  has a relative maximum at  $(x_0, y_0, z_0)$ , then  $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$ .

(b) **True or False:** The vector-valued function  $\langle 4 \cos t, 4 \sin t, t \rangle$  lies on the cylinder  $x^2 + y^2 = 16$ .

(c) **True or False:** If  $\mathbf{r}(t)$  and  $\mathbf{u}(t)$  are differentiable vector-valued functions, then  $\frac{d}{dt} [\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}'(t) \cdot \mathbf{u}'(t)$ .

(d) **True or False:** If the second-order partial derivatives of  $f$  exist at  $(a, b)$ , then  $f_{xy}(a, b) = f_{yx}(a, b)$ .

4. (8 points) Find the curvature  $K$  of the curve

$$\mathbf{r}(t) = 4 \cos t \mathbf{i} + 3 \sin t \mathbf{j} + t \mathbf{k}$$

at the point  $(-4, 0, \pi)$ .

5. (10 points) (a) Calculate the limit:  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^3 - y^3}$ .

(b) Discuss the continuity of the function  $g(x, y)$ . Justify your answer.

$$g(x, y) = \begin{cases} \frac{x^2 + 2xy^2 + y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases}$$

6. (8 points) To determine the height of a tower, the angle of elevation to the top of the tower is measured from a point  $100 \pm \frac{1}{2}$  foot from the base. The angle is measured at 0.6 radians, with a possible error of 0.02 radians. Assuming that the ground is horizontal:
- (a) For any angle  $\theta$  and distance  $d$ , use trigonometry to find an expression for the height  $h(d, \theta)$  of the building.
  - (b) Use differentials to measure the maximum error in determining the height of the tower.

7. (10 points) The temperature at the point  $(x, y)$  on a metal plate is

$$T = \frac{x}{x^2 + y^2}$$

- (a) Find the direction of greatest increase in heat from the point  $(3,4)$ .
- (b) If you are at the point  $(3,4)$ , how fast is the temperature changing in the direction of the origin  $(0,0)$ ? Is it increasing or decreasing?



8. (8 points) Let  $f(x, y) = ye^{xy}$  and  $P = (0, 2, 2)$ .
- (a) Find an equation for the tangent plane to  $f$  at  $P$ .
  - (b) Find parametric equations to represent the normal line to  $f$  at  $P$ .

9. (8 points) Calculate  $\partial w/\partial r$  and  $\partial w/\partial \theta$  if

$$w = x^2 - 2xy + y^2, \quad x = r + \theta, \quad \text{and } y = r - \theta.$$

**Extra Credit**(2 points) If you wanted to visualize the function

$$f(x, y, z) = 3x^2 - 4yz + z^2x^3,$$

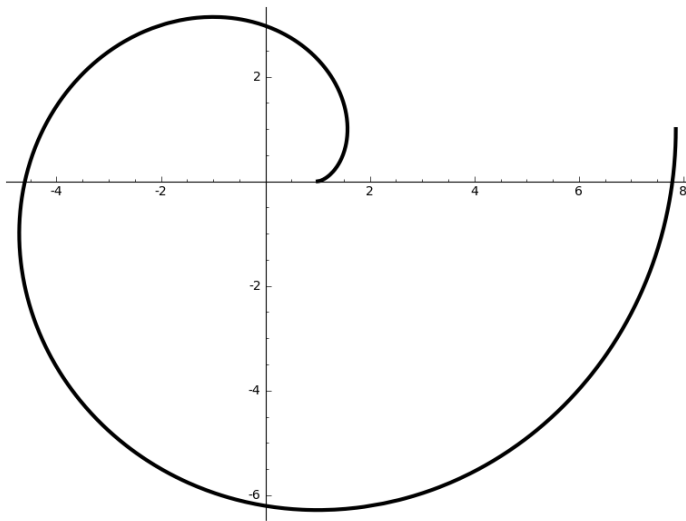
how would you do it? Explain in words and/or math notation.

10. (10 points) Let a particle move along the path

$$\mathbf{r}(t) = \langle \cos(\pi t) + \pi t \sin(\pi t), \sin(\pi t) - \pi t \cos(\pi t) \rangle.$$

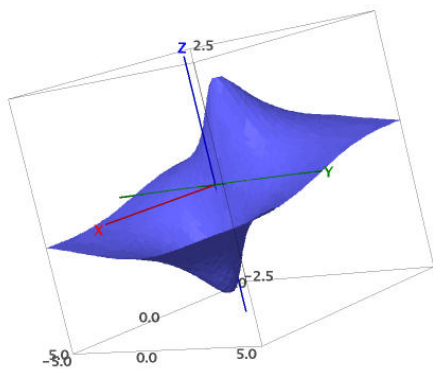
(a) Find expressions for  $\mathbf{T}(t)$  and  $\mathbf{N}(t)$ . Hint: The derivatives are a little tricky, but some things should cancel out and when you calculate the norm of  $\mathbf{r}'(t)$  it should simplify quite a bit.

(b) A graph of  $\mathbf{r}(t)$  is given below. Find the position for the particle at  $t = 2$ , and sketch the vectors  $\mathbf{T}(2)$  and  $\mathbf{N}(2)$  with their initial points at the position given by  $t = 2$ .

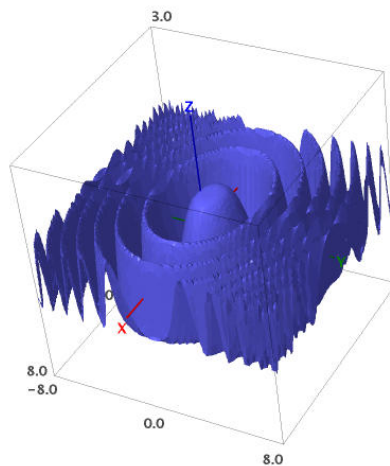


If you need more room for the last question, you can write here.

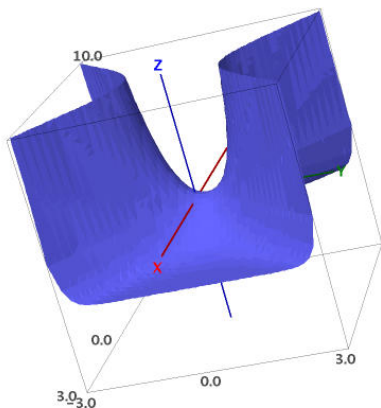
11. (8 points) Match the function on this page with the graph of its level curves on the next page.



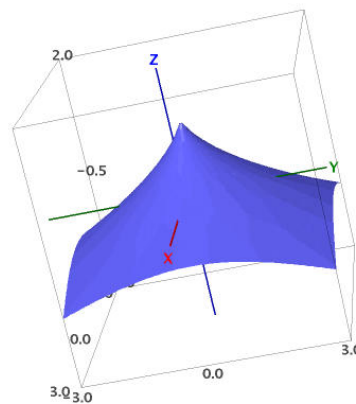
(a)  $f(x, y) = \frac{-4x}{x^2 + y^2 + 1}$



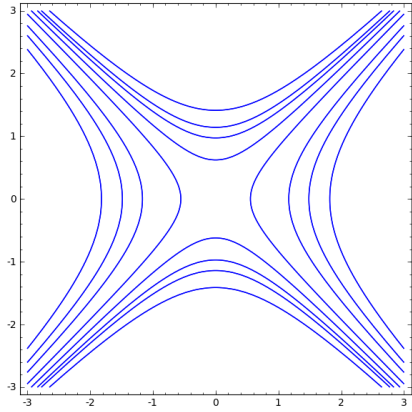
(b)  $f(x, y) = \cos\left(\frac{x^2 + 2y^2}{4}\right)$



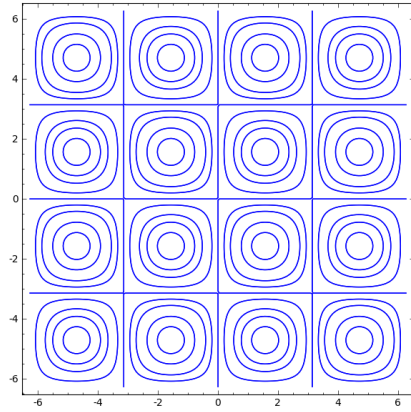
(c)  $f(x, y) = e^{1-x^2+y^2}$



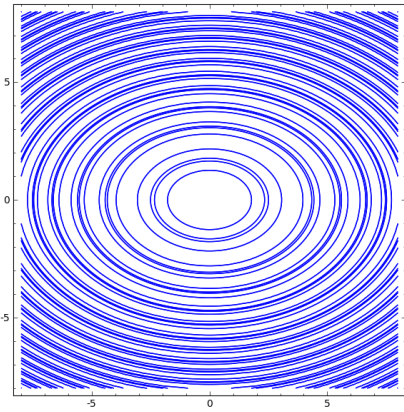
(d)  $f(x, y) = 1 - (x^2 + y^2)^{1/3}$



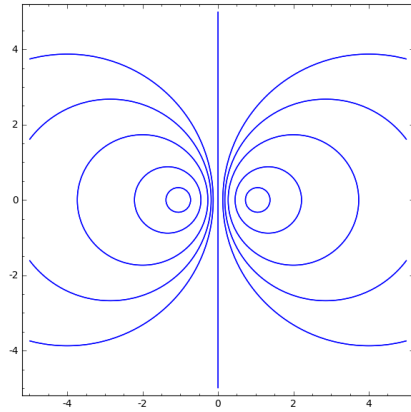
(e)



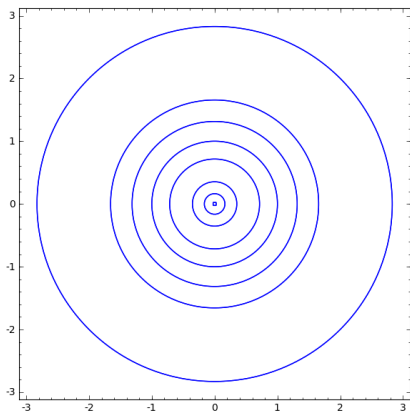
(f)



(g)



(h)



(i)