Minitest 4 - MTH 2410

Dr. Graham-Squire, Fall 2012

8:31
8:52
21

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. ~~Computers are allowed on part of this test, the very last question~~. No computers. Calculators are allowed on all ~~other~~ parts of the test. Even on questions where technology is allowed, you should still show all of your work.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 8. Total Points = 45.

1. (5 points) TRUE OR FALSE. Circle the correct answer. If false, give a counterexample or explain (briefly) why it is false. If true, no explanation is necessary (though if you are wrong, an explanation can get you some partial credit).

(a) **True or False:** If C is given by $x(t) = t$, $y(t) = t$, $0 \le t \le 1$, then

$$\int_C xy\, ds = \int_0^1 t^2\, dt$$

$x'(t)^2 + y'(t)^2$

Should be $\int_0^1 t^2 \sqrt{2}\, dt$

(b) **True or False:** If you calculate a surface integral of the function 1 over the surface S, you get the surface area of S.

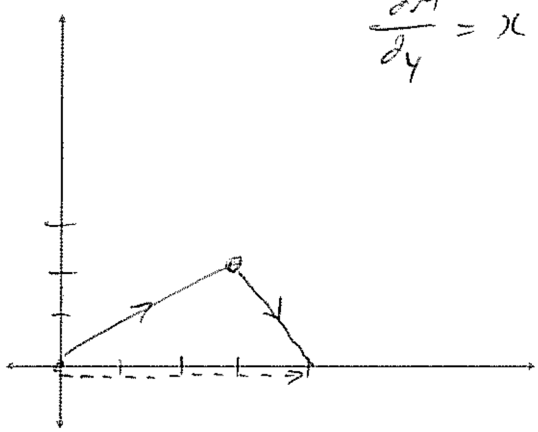
$$\iint f(x,y) \underbrace{\sqrt{1 + g_x^2 + g_y^2}}_{\text{surface area}}\, dA$$

For questions 2 to 5, use any appropriate technique to find the value of the given line integral. There will often be multiple ways to get to the correct answer, though one way is usually easier than the others. Make sure to explain your reasoning and show your work!

2. (6 points) Evaluate the line integral where C consists of the line segments from (0,0) to (3,2) and from (3,2) to (4,0).

$$\int_C \overbrace{(xy-1)}^{M}\,dx + \overbrace{(\tfrac{1}{2}x^2 - 2y + \cos(y))}^{N}\,dy$$

$$\frac{\partial M}{\partial y} = x \qquad \frac{\partial N}{\partial x} = x$$



$\Rightarrow$ conservative

$\Rightarrow$ can go on path directly

$r(t) = \langle t, 0 \rangle \qquad 0 \le t \le 4$

$r'(t) = \langle 1, 0 \rangle$

$F = \langle xy-1, \tfrac{1}{2}x^2 - 2y + \cos y \rangle = \langle -1, \tfrac{1}{2}t^2 + 1 \rangle$

$$\int_0^4 F \cdot r'(t)\,dt = \int_0^4 -1\,dt = \boxed{-4}$$

3. (6 points) Find the work done by the force field $\mathbf{F}(x, y, z) = \mathbf{i} + z\,\mathbf{j} + y\,\mathbf{k}$ when a particle moves along the path defined by $x = \cos t, y = \sin t$, and $z = t^2$ where $0 \le t \le \pi$.

$r(t) = \langle \cos t, \sin t, t^2 \rangle$, $r'(t) = \langle -\sin t, \cos t, 2t \rangle dt$

$F = \langle 1, t^2, \sin t \rangle$

$x + yz$

$$\int_0^\pi F \cdot dr = \int_0^\pi \left(-\sin t + t^2 \cos t + 2t \sin t \right) dt$$

✓✓✓✓

ugh ↗

F is conservative b/c $f = x + yz$ works.

start point $= r(0) = \langle 1, 0, 0 \rangle$

end point $= r(\pi) = \langle -1, 0, \pi^2 \rangle$ ✓

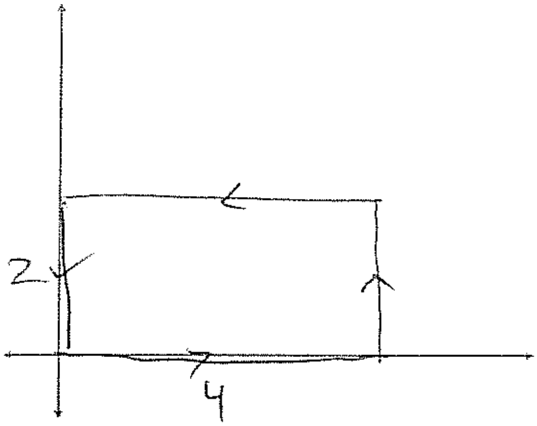
Use Fund. theorem:

$$\int_0^\pi F \cdot dr = x + yz \Big|_{\langle 1,0,0 \rangle}^{\langle -1, 0, \pi^2 \rangle}$$ ✓

$$= -1 + 0 - (1 + 0)$$

$$= \boxed{-2}$$

4. (6 points) Evaluate the line integral $\int_C (3x^2 - 2y)\,dx + (3x - y^2\sin(y))\,dy$ over the curve C given by the rectangle of height 2 and base 4, with its base on the positive x-axis and its left side on the positive y-axis.



closed curve

$\Rightarrow$ can use Green's theorem

$$\int_C \overset{M}{(3x^2-2y)}\,dx + \overset{N}{(3x-y^2\sin y)} = \int_0^4\int_0^2 \overset{\frac{\partial N}{\partial x}}{(3)} - \overset{\frac{\partial M}{\partial y}}{(-2)}\,dy\,dx$$

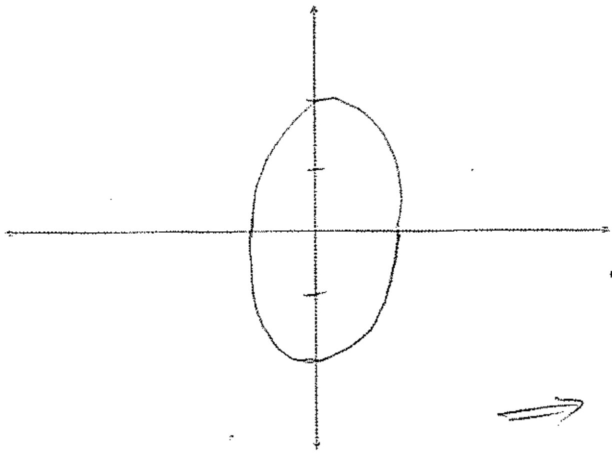
$$= 5\underbrace{\int_0^4\int_0^2 dy\,dx}_{=\text{ Area of rect.}}$$

$$= 5\cdot 8$$

$$= \boxed{40}$$

5. (6 points) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle \overset{M}{e^x + 6xy}, \overset{N}{\sin y + 3x^2} \rangle$ and C is given by

$$\mathbf{r}(t) = \cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}. \qquad 0 \le t \le 2\pi$$



$\frac{\partial M}{\partial y} = 6x$

$\frac{\partial N}{\partial x} = 6x$ $\Rightarrow$ conservative

and $\vec{r}(t)$ is a loop

$\Rightarrow$ By Green's theorem (or by fundamental theorem) the line integral will be $\boxed{0}$

7. (4 points) Match the equation to the graph.

(a) $\mathbf{r}(u,v) = \langle 4\cos(u)\cos(v), 4\sin(u)\cos(v), 4\sin(v)\rangle$ (ii)

(b) $\mathbf{r}(u,v) = \langle u+2, (v+u)/4, v\rangle$ (iv)

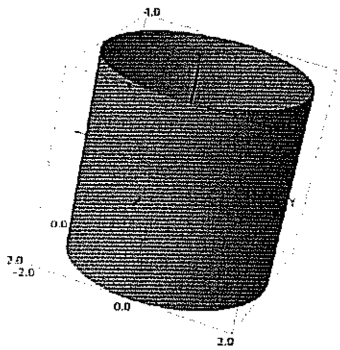
(c) $\mathbf{r}(u,v) = \langle 2\cos(u), 2\sin(u), v\rangle$ (i)

(d) $\mathbf{r}(u,v) = \langle (2+\cos(u))\cos(v), (2+\cos(u))\sin(v), \sin(u)\rangle$ (iii)

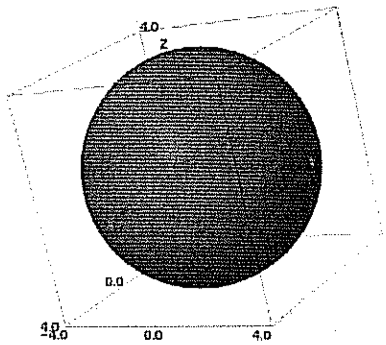
can check that

$x^2+y^2+z^2=16$

(c)

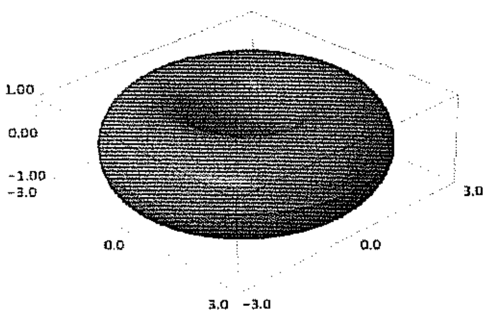


(i)



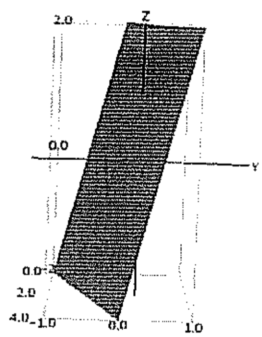
(ii)

(a)



(iii)

(d)

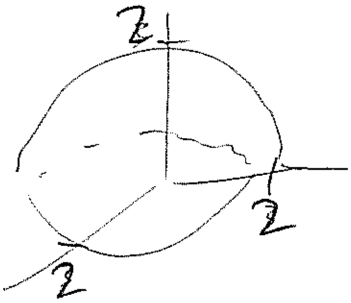


(iv)

(b)

8. (6 points) Evaluate the surface integral $\iint_S z\,dS$, where S is the hemisphere of radius 2 lying above the xy-plane.

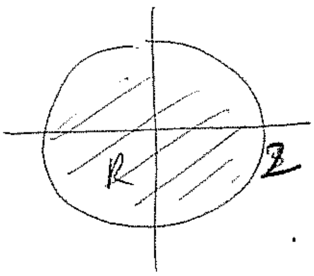
$$x^2+y^2+z^2=4$$



$$\Rightarrow z=\sqrt{4-x^2-y^2}=g(x,y)$$

$$g_x=\frac{-x}{\sqrt{4-x^2-y^2}} \qquad g_y=\frac{-y}{\sqrt{4-x^2-y^2}}$$

$$\Rightarrow \iint\left(\sqrt{4-x^2-y^2}\cdot\sqrt{1+g_x^2+g_y^2}\right)dA$$



$$=\iint\sqrt{4-x^2-y^2}\cdot\sqrt{1+\frac{y^2+x^2}{4-x^2-y^2}}\,dA$$

$$=\iint\sqrt{4-x^2-y^2}\cdot\sqrt{\frac{4}{4-x^2-y^2}}\,dA$$

$$=\iint 2\,dA$$

$$=\int_0^{2\pi}\int_0^{2} 2r\,dr\,d\theta=\int_0^{2\pi} r^2\Big|_0^2\,d\theta$$

$$=2\pi(4)=8\pi$$

Extra Credit(1 point) If $\int_C \mathbf{F}\cdot d\mathbf{r}=0$, then $\mathbf{F}$ and $d\mathbf{r}$ are orthogonal. (True or False)

Not true. Is true if $F\cdot dr=0$, but the integral can be zero even if F and dr are not always orthogonal.