Calculus III, MiniTest 4 Review

Dr. Graham-Squire, Fall 2012

•The test will cover sections 15.1-15.6.

•To study, you can look over your notes, rework HW problems on WebAssign, quizzes, and problems from the notes, as well as work out the practice problems given for each section. The Review Questions at the end of Chapter 15 are also good practice.

•Calculators <u>are</u> allowed on this test, but for certain questions you may not be allowed to use a calculator. It is highly recommended that you bring a calculator because you cannot use cell phones or computers during most of the test. I have not decided yet if there will be a question where you can use a computer.

•Some practice problems to work on are below. Note that for some of the integrals, it may be necessary to change the limits of integration or switch to polar, cylindrical, or spherical coordinates. Similarly, if given a line integral it may be easiest to compute it directly, using the fundamental theorem, or using Green's theorem. It is up to you to pick the correct method that is easiest for you to do.

1. Find the curl of the vector field $\mathbf{F}(x,y) = \dfrac{yz\mathbf{i} - xz\mathbf{j} - xy\mathbf{k}}{y^2z^2}$. Is $\mathbf{F}$ conservative? If so, find f such that $\nabla f = \mathbf{F}$.

2. Evaluate the line integrals:

   (a) $\displaystyle\int_C (2x-y)dx + (x+2y)dy$ where C is given by:

   (i) C: one revolution counterclockwise around the circle $x = 3\cos t$, $y = 3\sin t$.

   (ii) C: the line segment from (0,0) to (3,-3).

   (b) $\displaystyle\int_C xy\,dx + \frac{1}{2}x^2\,dy$, where C is the boundary of the region between the graphs of $y = x^2$ and $y = 1$.

   (c) $\displaystyle\int y\,dx + x\,dy + \frac{1}{z}\,dz$ where C is the curve $\mathbf{r}(t) = \langle t, t^2 - 3t, \frac{3}{4}t + 1\rangle$, $0 \le t \le 4$.

   (d) $\displaystyle\int_C (x^2 - y^2)\,dx + 2xy\,dy$, where C is given by $x^2 + y^2 = a^2$ (a is some constant).

   (e) $\displaystyle\int_C xy\,ds$ where C is the line segment from (0,0) to (5,4).

3. Find an equation for the tangent plane to the paraboloid given by

$$\mathbf{r}(u,v) = u\mathbf{i} + v\mathbf{j} + (u^2 + v^2)\mathbf{k}$$

   at the point (1,2,5).

4. Evaluate the surface integral $\iint_S z\,dS$ over the surface given by

$$\mathbf{r}(u,v) = (u+v)\mathbf{i} + (u-v)\mathbf{j} + \sin v\mathbf{k}$$

   where $0 \le u \le 2$ and $0 \le v \le \pi$. You may have to use Sage/Maple to evaluate the integral.